TESSEL WHAT?
MATH with a TWIST of ART!

6-8th Grade Lesson 8-4-16

By Tealecia Fletcher

PATTERN/TESSELLATION CURRICULUM UNIT INTRODUCTION

Why are the skills, content, and concepts presented in this unit important for students to learn and why do students need to understand the math behind them? An online article from Math Work Sheet Center said it best, called 10 Reasons Why It Is Important To Understand Mathematical Patterns and it states, "Wouldn't it be great if you could predict the future? Well, some people believe that predicting the future is impossible but it would be more accurate to say that making outlandish predictions not based in logic leads to low accuracy. However, looking at the relationship of a series of patterns over time can lead to making accurate predictions of particular results. This is a common method of mathematical pattern analysis and such an analysis is important for the following reasons:

1. Understanding mathematical patterns allows someone to identify such patterns when they first appear. After all, you cannot gain the benefit of patterns if you can't see them and you can only see them if you understand them.

2. Patterns provide a sense of order in what might otherwise appear chaotic. When you notice that things happen in a certain pattern - even something as mundane as a bus always stopping at a certain corner at 5pm - order is provided.

3. Patterns allow someone to make educated guesses. Much science is based on making a hypothesis and hypotheses are often based on understanding patterns. Similarly, we make many common assumptions based on recurring patterns.

4. Understanding patterns aid in developing mental skills. In order to recognize patterns one needs to have an understanding of critical thinking and logic and these are clearly important skills to develop.

5. Patterns can provide a clear understanding of mathematical relationships. This can be seen in a very evident manner in the form of multiplication tables. 2 x2, 2 x 4, 2 x 6 are clearly examples of the relationship pattern found in multiplication.

6. Understanding patterns can provide the basis for understanding algebra. This is because a major component of solving algebra problems involves data analysis, which is deeply related to the understanding of patterns. Without being able to recognize the appearance of patterns the ability to be proficient in algebra will be limited.

7. Understanding patterns provide a clear basis for problem solving skills. In a way, this is related to critical thinking but more directed towards mathematics specifically. Patterns essentially provide a means of recognizing the broader aspects that can be shored down in order to arrive at the specific answer to a particular problem.

8. Knowledge of patterns is transferred into science fields where they prove very helpful. Understanding animal patterns has been used to help endangered species. Understanding weather patterns not only allows one to predict the weather but also predict the common impact of weather, which can aid in devising the appropriate response in an emergency situation.

9. One of the lesser-known aspects of patterns is the fact that they often form the basis of music. For example, there are various patterns of notes that provide the

basis for proper harmony on a piano. If you don't believe patterns are important when playing a piano simply walk up to the nearest piano and start banging away randomly at the keys. You probably won't hear any songs that you recognize!

10. Patterns provide clear insight into the natural world. While animals and certainly plants are far from thinking beings they do have certain habits that exist in patterns and understanding these behavioral patterns provide a clearer understanding of all living things."[1]

The National Council of Teachers of Mathematics also recognizes the importance of patterns. In its publication, *Curriculum and Evaluation Standards for School Mathematics* (1989), they conclude that, "Patterns abound in our world. The mathematics curriculum should help sensitize students to the patterns they meet every day and to the mathematical descriptions or models of these patterns and relationships."[2] In its discussion draft of **Principles and Standards for School Mathematics** (1998), the NCTM has moved further to place the Standard of Patterns at every grade level from preK to 12. Placing pattern standards at every grade level emphasizes just how important professionals feel it is.

Patterns pervade every part of our daily live. The concepts and properties of pattern further help young minds explore and construct their own understanding of the complexities in our world. Understanding pattern helps with any real-life math that requires critical thinking skills, problem solving and logical reasoning. It is my hope that students taught this unit, will have a better understanding of how the world is structured around principles of pattern. Using examples from real life and connecting pattern ideas to their own personal history, will better help explain and clarify how powerful pattern really is.

Special features are included in the unit. There are 4 evidence-based instructional strategies to differentiate for gifted students, including Taba, Visual Thinking Strategy, Bruner and Questioning. These address all the features of differentiation by offering various levels of complexity, challenge, depth, creativity and acceleration.

The Taba Lesson is very open-ended and allows students to pull from their personal knowledge and background as they tour the school seeking patterns. As they list, group, label, regroup, subsume, generalize, summarize and synthesize, students are challenged to examine the concept of pattern very deeply. The resulting discussions are powerful, complex, enriching and encourage strong understanding and internalization. As the teacher observes and listens, this makes it very easy to assess understanding and to what degree.

The Visual Thinking Strategy Lesson again allows students to make it personal as they discuss what they see or interpret in an art piece. An approach like this lets gifted students flourish as their interpretations reflect them and any direction they want to take. It encourages description and/or explanation to ensure that the students are able to give full, thoughtful answers. This lesson then has students explore pattern around the world in a real world context. There are station activities so that each "country or continent" they visit, gets them to do a short exploration through guided questions, of a specific type of pattern called tessellations. They even get to color, accessing their creativity, which gifted students love! Lastly, students get to explore a variety of artists that use unexpected materials to enhance their creativity and increase engagement.

The Bruner Lesson is great for student development and retention of the pattern concept knowledge by having the student approach the learning through an artist's point of view. Putting themselves in the artist's shoes and having to pick a mentor also creates a fun, engaging learning environment. This requires students to use strategy, prior knowledge from the unit so far, and their strong opinions and judgment skills, which is also very complex and challenging.

The Questioning Lesson has lots of applications of the knowledge and skills students have gained in the unit so far. The mission control activity tests their ability to communicate these clearly, while the envelope activity has them extend these to greater heights, through inquiry. This lesson achieves differentiation through the questioning and the discussion of answers.

The population of gifted children for whom this unit is intended is those who work well both independently and in groups. Students should be high achievers with an exceptional level of self-motivation and skilled at making connections. This unit is particularly successful for visual and tactile learners who should have a strong background in basic geometry and logic. Multicultural backgrounds, multi-socioeconomic backgrounds and varied ages should add depth and variety to the classroom climate.

I feel that the 4 various learning methods can only help to encourage the development and progression of all learners. Such a diverse range helps to recognize, value and respond to individual student need, motivation, experience, prior knowledge and aspirations. There are many important reasons for students to understand the math taught in this unit. The math learned here applies to student courses now, future courses, cross curriculum and to their current and future lives.

Resources

1. 10 Reasons Why It is Important to understand Mathematical Patterns?
http://www.mathworksheetscenter.com/mathtips/mathpatterns.html

2. Principles and Standards for School Mathematics
http://www.nctm.org/standards/

CONTENT, PROCESS, AND CONCEPT GOALS

CONTENT GOALS AND OUTCOMES

Goal 1: To develop understanding of the key elements of pattern including application to math and art.

Students will be able to…

A. Define pattern as a repeating unit of shape or form and in math/geometry it's additionally things arranged following a rule.
B. Identify, describe, compare, contrast and categorize patterns in relationship to each other.
C. Identify tessellations, a specific pattern, using 3 requirements.
D. Communicate about details of specific types of patterns, using planes, tilings, polygons, vertices, and degree measures.
E. Give examples of and explain math connections to art.

PROCESS GOALS AND OUTCOMES

Goal 2: To develop reasoning skills with how patterns informs predictions.

Students will be able to…

A. Predict the next item or part of a pattern, given a starting portion.
B. Apply and transfer pattern content knowledge to define and solve real world problems.
C. Create and critique art using pattern knowledge and skills.
D. Enhance their language, critical and creative thinking skills from identifying objects, people and actions in art.

CONCEPT GOALS AND OUTCOMES

Goal 3: To understand the concept of pattern.

Students will be able to…

A. Use math/geometry language to identify the basic elements of pattern, like solid versus print, same number versus different number, number order or shape.
B. Make the inference that math is basically the study of pattern
C. Recognize that pattern is everywhere and pattern integrates math with other curriculum topics like music.
D. Provide evidence that pattern can be more than just simple repetition.
E. Analyze how patterns are used.
F. Internalize that studying pattern is an opportunity to observe, hypothesize, experiment, discover and create.

ASSESSMENT PLAN

Performance Task(s)

DAY 1/ PART 1

(70 min.) Teacher introduces the skills portion of the performance task evaluation for the unit and STUDENTS CREATE THE 1ST PHASE OF THEIR TESSELLATION, MAKING A TRANSLATION TESSELLATION

(20 min.) The teacher states, a tessellation is a **type** of pattern. It's made of repeating shapes that cover a surface completely without overlapping or leaving any gaps. Covering the surface completely is also called tiling the plane. Basically 3 rules for Tessellations:

1. Repeating Shapes
2. Tile the plane
3. No gaps or overlaps

A checkerboard is a tessellation made of squares. The squares meet edge to edge with no gaps and no overlapping areas. The pattern of bricks on a wall is a tessellation made of rectangles.

Over 2,200 years ago, ancient Greeks were decorating their homes with tessellations, making elaborate mosaics from tiny, square tiles. Early Persian and Islamic artists also created spectacular tessellating designs. More recently, the Dutch artist M.C. Escher used tessellations to create enchanting patterns of interlocking creatures, such as birds and fish. We will examine this more in depth tomorrow.

Making tessellations combines the creativity of an art project with the challenge of solving a puzzle. Let's look at a basic skill required to make tessellations.

Lesson on tessellations using cool math site online (pattern blocks, teacher pauses for students to experiment and make conclusions at each step)

http://www.coolmath.com/lesson-tessellations-1

(45 min.) Teacher explains what the final performance task will be and that it's skills will be practiced each day.

Performance Task: Commissioned Artist Activity explained:

A very prestigious art museum is having a contest for the cover art for their next advertising campaign that includes a brochure, posters and newspapers. This is a great opportunity for new artists like you to get exposure to increase your sales and chances for success. You are challenged to meet the requirements. The winning piece will be the one that is most representative of the theme and meets all requirements. The theme is pattern and the museum committee will be judging all entrants on the following criteria:

4. *Artist must include the written plan they followed to create their work of art.*
5. *Artist must make use of several types of patterns, including the three types of tessellations and at least one special pattern like Fibonacci or the Golden Ratio.*
6. *Artist must include a description of the inspiration for their art.*
7. *Artist must include a connection to nature and/or the real world.*
8. *Artist is challenged to be creative with use of pattern.*
9. *Art must be original.*

Teacher led, STUDENTS CREATE THE 1ST PHASE OF THEIR TESSELLATION, MAKING A TRANSLATION TESSELLATION (see attached)

(5 min.) clean up of materials

DAY 2/ PART 2

(35 min.) STUDENTS CREATE THE 2ND PHASE OF THEIR TESSELLATION EVALUATION, REFLECTING A TESSELLATION (see attached)

DAY 3/ BEGIN PART 3

(50 min.) ***STUDENTS CREATE THE 3rd PHASE OF THEIR TESSELLATION EVALUATION, ROTATING A TESSELLATION (see attached) Student are also encouraged to add features and designs to these tessellations.***

DAY 4/END PART 3

(15 min.) Honing skill activity. Teacher led, STUDENTS COMPLETE CREATING THE 3rd. PHASE OF THEIR TESSELLATION, MAKING A ROTATION TESSELLATION and helps students review their tessellation skills from the week. (see attached)

DAY 4 ART CREATION

Students create their art for the Performance Task: Commissioned Artist Activity

(30 min.) **Evaluate:** *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*

Teacher arranges student artwork around the room. Teacher tells students they are tasked to act as the committee judging the artwork and go around the room making detailed observations. After observations of each piece of work, the teacher makes student groups of no more than 3 students per group who then meet to record their findings and make their generalizations. The teacher then provides time for groups to have one final whole class meeting to draw conclusions based on all the observations. The teacher asks students to report their findings. Each group reports their conclusions and reasons for their conclusions. Students are encouraged to discuss conclusions of other groups and to contribute by questioning and adding their own conclusions. Finally, the class is tasked to award each piece of art a category, no duplicates – each art piece may be awarded one category.

Committees meet and decide the following awards:

1. *Best use of pattern.*
2. *Best use of color.*
3. *Best-written submission.*
4. *Best connection to nature/ the real world.*
5. *Best use of tessellations.*
6. *Most symmetrical.*
7. *Most Entertaining.*
8. *Most Interesting. (Makes you think)*
9. *Best use of a variety of media.*
10. *Most original.*
11. *Most creative.*
12. *Best interpretation of the pattern theme.*
13. *Quality of composition/work.*
14. *Best overall clear emphasis of the theme pattern.*

- *Each day of the activity, the teacher is observing and assisting students. The teacher asks targeted questions to help students gain understanding and confidence in their ability to create tessellations by translating, reflecting and rotating as well as combinations of the three for more advanced students.*

COPYING/PASTING DID NOT INCLUDE THE DIAGRAMS, I WILL ATTACH THE ORIGINAL.

EXPLORING TESSELLATIONS

Activities | Grades 6–8

www.exploratorium.edu/geometryplayground/activities

Background: What is a tessellation?

A tessellation is any pattern made of repeating shapes that covers a surface completely without overlapping or leaving any gaps. A checkerboard is a tessellation made of squares. The squares meet edge to edge with no gaps and no overlapping areas. The pattern of bricks on a wall is a tessellation made of rectangles.

Over 2,200 years ago, ancient Greeks were decorating their homes with tessellations, making elaborate mosaics from tiny, square tiles. Early Persian and Islamic artists also created spectacular tessellating designs. More recently, the Dutch artist M. C. Escher used tessellation to create enchanting patterns of interlocking creatures, such as birds and fish.

Making tessellations combines the creativity of an art project with the challenge of solving a puzzle.

Part One: Making a Translation Tessellation

[45 minutes]

Suppose you wanted to cover a floor with tiles. You could cover it with square tiles, since squares fit together without leaving any gaps.

In this activity, you're going to transform a rectangle into a more interesting shape, then make a tessellation by repeating that shape over and over again.

Materials:

- Index card 3" x 5"
- Ruler
- Scissors
- Blank paper
- Pencil
- Transparent tape
- Colored markers or pens
- 2.5" x 3" Grid paper (included)
- Optional: Printed example of a tessellation to show students before starting activity. (Possibly from the online M. C. Escher gallery at: http://www.mcescher.com/Gallery/gallery-recogn.htm)

Try This:

Step 1 Cut an index card in half, creating a 2.5" x 3" rectangle.

Step 2 Find the area of the rectangle (length x width).

Step 3 Draw a line between two adjacent corners on one of the long sides of the rectangle. Your line can be squiggly or made up of straight segments. Whatever its shape, your line must connect two corners that share one side of the rectangle.

Step 4 Cut along the line you drew. Take the piece you cut off and slide it straight across to the opposite long side of the rectangle. Line up the long, straight edges of the two pieces and tape them together.

Step 5 Can you tessellate with this shape? Try tracing this shape several times, creating a row going across a

piece of paper. Line up the cut edges of the shape as you trace it.
TAPE

Step 6 Now draw another line that connects two adjacent corners on one of the short sides of the shape.
Step 7 Cut along this new line. Take the piece you cut off and slide it straight across to the opposite side of the shape. Line up the straight edges and tape them together.
Step 8 You have now created a shape that you can use as a pattern to make a tessellation.
What's the area of this shape? Write the letter A on one side of the shape and turn it over and write the letter B on the other side.
Step 9 On your grid paper, carefully trace around your pattern shape.
Can you figure out where to place the pattern so that your paper will be covered with repetitions of this shape with no overlaps and no gaps? Try to cover your whole sheet of paper by tracing the pattern, moving it, then tracing it again.
If you start with side A facing up do you ever have to turn it over to side B to make your tessellation? If you only have to slide the piece without flipping it over or rotating it, then you are making a translation tessellation.
In math, translation means shifting the position of a shape without moving it in any other way.
TAPE
TAPE
TAPE

Step 10 Look for a clever way to color in the resulting design on your sheet of paper.
Does your shape look like a fish? A bird? An elephant?

Part Two: Making a Different Kind of Tessellation

[30 minutes]

You've made translation tessellations by tracing a pattern, then sliding it to a new position and tracing it again. Now you can make a different kind of tessellation by turning (rotating), or by flipping over (reflecting) the pattern to a new position before copying it again.

Materials:

- Index card, 3" x 5"
- Ruler
- Scissors
- Blank paper
- Pencil
- Transparent tape

• Colored markers or pens
• 2.5" x 3" Grid paper (included)

Try This:

Step 1 Cut an index card in half, creating a 2.5" x 3" rectangle.

Step 2 Draw a line between two adjacent corners on one of the long sides of the rectangle. Your line can be squiggly or made up of straight segments. Whatever its shape, your line must connect two corners that share one of the long sides of the rectangle.

Step 3 Cut along the line you drew. Take the piece you cut off, flip it over and then slide it across to the opposite long side of the rectangle. Line up the straight edge of the piece with the straight edge of the opposite edge of the rectangle. Tape the piece in place.

TAPE

Step 4 Now draw another line that connects two adjacent corners on one of the short sides of the shape.

Step 5 Cut along this new line. Take the piece you cut off, flip it over and then slide it straight across to the opposite side of the shape. Line up the straight edge of the piece with the straight edge of the shape. Tape the piece in place.

Step 6 You have created a shape that you can now use as a pattern to make a tessellation. Write the letter A on one side of the pattern, then turn it over and write the letter B on the other side.

Step 7 On your grid paper, carefully trace around your pattern shape. It may help to position the squared-off corner (formerly the edge of the index card) in one corner of the grid.

Can you figure out where to place the pattern piece so that your paper will be covered with repetitions of this shape with no overlapping and with no gaps?

Try to cover your whole sheet of paper by tracing the pattern, moving it, then tracing it again.

If you start with side A facing up, do you ever have to turn it over to side B to make your tessellation?

If you have to flip your piece over, you are making a reflection tessellation. If you also had to move the piece to a new position, you have used translation.

Step 8 Look for a clever way to color in the resulting design on your sheet of paper.

TAPE
TAPE
TAPE

When you cut a shape out of paper, then flip it over, the flipped shape looks like a mirror image of the original shape. So a tessellation made with this technique is called a reflection tessellation.
Your hands can help you understand the concept of mirror reflection. Your two hands are the same shape—but your right hand is a mirror reflection of your left hand (and vice versa.)

Part Three: Making a Rotation Tessellation

[30 minutes]
Materials:
- Index card
- Ruler
- Scissors
- Blank paper
- Pencil
- Transparent tape
- Colored markers or pens
- 2.5" Square Grid paper (included)

Try This:
Step 1 Draw a 2.5" x 2.5" square on your index card.
Step 2 Cut out the square from the index card.
Step 3 Draw a line between two adjacent corners on one side of the square. Your line can be squiggly or made up of straight segments. Whatever its shape, your line must connect two corners that share one side of the square.
Step 4 Cut along the line you drew. Take the piece you cut off (without flipping) and slide it to an adjacent side of the square. Line up the straight edges and tape them together.
TAPE

Step 5 Now draw another line that connects the two corners on the side adjacent to the cut side of the square.
Step 6 Cut along this new line. Take the piece you cut off (without flipping) and slide it to its adjacent side. Line up the straight edge of the cut piece with the straight edge of the square, and tape them together.
Step 7 You have now created a shape you can use as a pattern to make a tessellation. Write the letter A on one side of the shape and turn it over and write the letter B on the other side.
Step 8 On your grid paper, carefully trace around your pattern piece.
Try to cover your whole sheet of paper by tracing the pattern, then moving it and tracing it again. If you start with the side A facing up, do you ever have to turn it over to side B to make your tessellation? If you have to flip your piece over, you have made a reflection tessellation. If you also had to move the piece to a new position

you have also used translation. If you have to turn or rotate the shape to make your tessellation, then you have made a rotation tessellation.

Step 9 Look for a clever way to color in the resulting design on your sheet of paper.

TAPE
TAPE
TAPE

What's Going On?

If you look around, you'll see many repeating patterns: on wallpaper, on fabric, in a tiled bathroom floor. In this activity, we explored some of the mathematical rules used to make repeating patterns. Tessellations are patterns that cover a surface completely without overlapping or leaving any gaps.

The three mathematical rules of repetition used in this activity are:

- Shifting the position of a shape (something mathematicians call translation.)
- Rotating a shape to a new position (mathematicians don't have a fancy name for this; they just call it rotation.)
- Flipping a shape over so it looks like a mirror reflection of itself (mathematicians call this reflection.)

Many people think of M. C. Escher's work when they think of tessellations. (http://www.mcescher.com/Gallery/gallery-recogn.htm) Escher worked for a long time on each of his designs before he was satisfied with the final result. After you try this activity once, you may want to make several more pattern pieces until you come up with an especially interesting or evocative design.

People don't usually think of math when they look at beautiful designs, but math provides tools that help us create and analyze patterns. The three rules used to create these tessellations—translation, rotation, and reflection—are used in making many of the repeating designs we see around us every day.

Going Further:

To view the work of M. C. Escher, go to:
http://www.mcescher.com/Gallery/gallery-recogn.htm

To see tessellations that other students have made, go to:
http://www.worldofescher.com/contest/

The site includes an ongoing contest for the best tessellation, and has a "Hall of Fame" that includes a number of tessellations by middle-schoolers.

2.5" x 3" Grid

2.5" Square Grid

National Education Standards | Grades 6–8

From the National Council of Teachers of Mathematics (NCTM)

EXPLORING TESSELLATIONS

Apply transformations and use symmetry to analyze mathematical situations:

- Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.

Use visualization, spatial reasoning, and geometric modeling to solve problems:

- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Apply appropriate techniques, tools, and formulas to determine measurements:
• Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.

TEACHER NAME	Lesson #
Tealecia Fletcher	1

MODEL	CONTENT AREA	GRADE LEVEL
Taba	Mathematics	6

CONCEPTUAL LENS	LESSON TOPIC
Patterns	Patterns/Tessellations

LEARNING OBJECTIVES *(from State/Local Curriculum)*

CCSS.MATH.CONTENT.K.G.1
DESCRIBE OBJECTS IN THE ENVIRONMENT USING NAMES OF SHAPES, AND DESCRIBE THE RELATIVE POSITION OF THESE OBJECTS.

CCSS.MATH.CONTENT.2.G.A.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles,

quadrilaterals, pentagons, and hexagons.

CCSS.MATH.CONTENT.4.G.A.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

CCSS.MATH.CONTENT.4.G.A.3

Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into

matching parts. Identify line-symmetric figures and draw lines of symmetry

G-CO Experiment with transformations in the plane.

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-GMD Visualize relationships between two-dimensional and three-dimensional objects.

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects.

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	THE ESSENTIAL QUESTION *(What question will be asked to lead students to "uncover" the Essential Understanding)*
Patterns Inform Predictions	***In what ways do patterns inform predictions?***

CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	PROCESS SKILLS (What will students be able to do as a result of this lesson?)
Students will know.. **1. That pattern is defined as a repeating unit of shape or form and in math/geometry it's additionally things arranged following a rule.** 2. **The basic elements of pattern, like solid versus print, same number versus different number, number order or shape.** **3. That math is basically the study of pattern.** **4. That pattern integrates math with other topics like music.** **5. That patterns are everywhere.** **6. That pattern can be more than just simple repetition.** **7. That patterns allow us to make predictions.** **8. That there are 3 requirements for a tessellation.** **9. That tessellating or tiling is covering a plane with no gaps or overlaps.** **10. Polygons have three or more sides, their sides are lines not curves and they are closed figures.** **11. Vertices are where the endpoints of sides of polygons meet.** **12. Regular Tessellations have regular polygons (same**	Students will be able to... 1. Identify patterns using detailed characterisics. 2. Draw conclusions and make generalizations about patterns. 3. Recognize patterns in the environment. **4. Identify** the elements of patterns using detailed characterisics. **5. Describe and categorize patterns in relationship to each other.** **6. Apply the concept of pattern to their life and the world around them.** **7. Recognize that studying pattern is an opportunity to observe, hypothesize, experiment, discover and create.** **8. Conclude that pattern applies to all areas of study.**

sides/same angles) and must be the same at every vertex. **13. Angles at every vertex must add up to 360 degrees.** **14. For polygons, only regular triangles, squares, rectangles, and hexagons, tessellate.** **15. Semi-regular tessellations have two or more regular polygons and also must be the same at every vertex.** **16. Semi-regular tessellations may have the same polygons, but different configurations.** **17. How to name or label tessellations using numbers.**	

GUIDING QUESTIONS

What questions will be asked to support instruction?
Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	During Lesson Questions:	Post Lesson Questions:
1. What is a pattern? **2. What are some other examples of patterns?** **3. What types of patterns have you experienced other than what you can see like pictures or objects?** **4. What types of patterns do you see around us right now in our classroom?** **5. What kind of predictions can we make with these patterns?**	**1. How can you label the groups you have formed?** **2. How did you choose where to place the items?** **3. How are the items similar, in relationship to pattern?** **4. How are the items different in relationship to pattern?** **5. Is there any item that could belong to more than one group?** **6. Could any group be subsumed (absorbed) by another?**	***1.*** **What kinds of pattern varieties are there?** **2. How do patterns affect or influence our environment/lives?** **3. Describe how patterns have value/worth to society, other than monetary?** ***4.*** **What can you say that is true about pattern that you did not know before?** ***5.*** **Using what you observed today, discuss nature patterns versus manmade patterns.** ***6.*** **How does your recognition and study of pattern help you understand the world around you?** ***7.*** **After our exploration, would you like to adjust your definition of pattern?** **8. What is the purpose of pattern?**

DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	Learning Environment
	Students engage in in-depth critical thinking as they must analyze the concept of pattern in great depth through grouping and regrouping.		

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect - *This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.*

Teacher introduces self and spells out name on the board. Teacher notes there is a student named Tea, we share names, cool!
Teacher requests that if students own ear buds, to please bring then throughout the week.
Teacher guides a discussion of class expectations:

1. *Respect each other*
2. *Only positive responses allowed even if you disagree, do so in a nice way*
3. *Teacher explains and demonstrates call & response refocus; Teacher says, Mic check mic check? Students say 1,2…*

(20 min.) 12:30 – 12:50 Ice Breaker:

1) Students will be randomly given a card from a Game called Set. The cards have 4 possible categories on them:
   A) Shape (diamond, oval or squiggle)
   B) Color (red, purple or green)
   C) Number of shapes (one, two or three)
   D) Pattern (no pattern or blank, stripe, or solid)

2) Students will be asked to hold their card in front of them so that others can see and then group themselves according to a characteristic of the item(s) on their cards, either all 3 cards the same characteristic or all 3 cards a different characteristic. Only 3 to a group and all cards must be used at least once. They must find at least 9 sets. The whole class will have to use strategy to ensure that all the cards are used. Students must record each group of 3 they find-using sketches on the board. No duplicate groups of 3 allowed.

3) When step 2 task is complete, the teacher will then give the class, question categories A and B. Teacher directs that all answers must be G rated/school appropriate, and gives each student a chance to share.

4) Students will then be asked to regroup themselves differently from the first time, using the same cards with an additional 9 cards, students may not duplicate any previously used groups of 3, and continue recording their 3 groups on the board as a reference. They must find at least 7 sets. When step 4 is successful, for category questions C and D, each group member will be given a few minutes to think about their answers prior to sharing.

CATEGORY QUESTIONS:

a) What's your grade and school?
b) What are your favorite hobbies/activities?
c) What's your favorite movie, television show, type of music or song?
d) Why did you choose this class?

Explore - *In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.*

(40 min.) 12:50 – 1:30 INTRODUCTION TO PATTERNS (and WALK):

(Students are provided with clip boards, lined paper or their composition notebooks, whichever they prefer and writing utensils)

1) CLASS DISCUSSION: (10 min.) Teacher states, "During our ice breaker, you explored a few basic characteristics of something we call pattern. Can anyone name and describe one of those characteristics? Teacher calls on students with hands raised to share, looking for answers, ***COLOR, SHAPE, PATTERN or PRINT AND NUMBER.***

Teacher asks the pre-lesson questions: **Throughout the pre-leeson questions, the teacher is adding the student responses to the board for all to see.**

1. **What is a pattern?**
2. **What are some other examples of patterns?**
3. **What types of patterns have you experienced other than what you can see like pictures or objects?**
4. **What types of patterns do you see around us right now in our classroom? (Students are given the opportunity to share anything related to pattern. Teacher evaluates responses and gently corrects if item is completely not a pattern or redirects to help the student make a more accurate response.)**
5. **What kind of predictions can we make with these patterns?**

The teacher sums up the statements made by students by stating I think we can all now agree that **pattern is defined** as a repeating unit of shape or form, and in math or geometry it's additionally things arranged following a rule.
Teacher states, we live in a universe of pattern. Mathematics is basically the study of patterns. Studying pattern is an opportunity to observe, hypothesize, experiment, discover and to create. By understanding regularities based on the data we gather we can predict what comes next, estimate if the same pattern will occur when variables are altered and begin to extend the pattern. Pattern is not always just simply about repetition!

The study of pattern integrates both the strands of mathematics and a variety of curricular areas. The teacher asks students to share what other topics patterns can apply to, and fills in any they miss. We can use and extend skills and knowledge of number, measurement, geometry, data collection and statistics, probability and algebraic thinking. It allows us to bring together mathematics with music, visual arts and crafts, vocabulary building, creative writing and verbal communication, social studies, science and environmental studies, talent and technology.
(Pattern Power Stage: 1, 2 and 3 Article by Dr Merilyn Buchanan Published October 2001,January 2004,February 2011.)

2) PATTERN WALK: (20 min.)

When teacher is satisfied the class has a clear understanding of what to look for, students are taken on a "walk" to explore the hallway, outside, or what areas are available and ask students to write down what they observe related to pattern and anything they can think of that has or is related to pattern.

3) LISTING: (10 min.)

Back in class, students will share their lists and the teacher will make a comprehensive list on the board.

Explain - *Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.*

5 min. break (50 min.) 1:30 -2:20 GROUPING AND LABELING:

(10 min.)

1. Students will create smaller word lists based on similarities with respect to pattern (with their assigned group of 3 students.) Groups will work together in order to decide which items in the comprehensive list go together because they are alike in some way with respect to pattern. Students will be guided to follow the following rules:
2. At least 4 different groups
3. At least 3 items in each group
4. Each item may only be used once, in one group, no duplicates.
5. The teacher will move throughout the classroom checking in with student groups. The teacher will guide students as necessary with questions, but will allow the student groups to come to their own conclusions. As students finish, the teacher will instruct students to label the newly defined groups with the clearest, most precise label for each of their groups, by asking, "Do any of these items belong together?" "How?" Students will explain their reasons. The teacher will ask students to describe the similarities and differences among the groupings.

(10 min.) The teacher asks the during the lesson questions:

Think about:

1. **How can you label the groups you have formed?**
2. **How did you choose where to place the items?**
3. **How are the items similar, in relationship to pattern?**
4. **How are the items different in relationship to pattern?**
5. **Is there any item that could belong to more than one group?**
6. **Could any group be subsumed (absorbed) by another?**

Elaborate —*Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways*

SUBSUMING, REGROUPING, RENAMING

(10 min.)

1. Student groups will be challenged to group the items again but in a different way, using new categories. Teacher will remind students that groupings must be based on some aspect of pattern. Rules for regrouping:
   a. Groupings must be new/different from what they used the first time
   b. Groupings must be based on similarities with regard to pattern
   c. Must have at least 3 groups
   d. Each group must have at least 3 items
   e. Items may only be used once, in one grouping, no duplicates

(10 min.)

2. The teacher will ask all groups to share their groupings. Class discusses if there were any groups that could be absorbed by others.
3. The teacher asks the post lesson questions:
   10. **What kinds of pattern varieties are there?**
   11. **How do patterns affect or influence our environment/lives?**
   12. **Describe how patterns have value/worth to society, other than monetary?**
   13. **What can you say that is true about pattern that you did not know before?**
   14. **Using what you observed today, discuss nature patterns versus manmade patterns.**
   15. **How does your recognition and study of pattern help you understand the world around you?**
   16. **After our exploration, would you like to adjust your definition of pattern?**

Evaluate: *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*

(5 min.) The teacher will wrap up the lesson by posing a targeted question to the groups:
What is the purpose of pattern?

17. ***The teacher will assign students to talk to family members about what they know about pattern for quick discussion Tuesday.***

(70 min.) 2:20 – 3:30 Teacher introduces the skills portion of the performance task evaluation for the unit and STUDENTS CREATE THE 1ST PHASE OF THEIR TESSELLATION, MAKING A TRANSLATION TESSELLATION

(20 min.) The teacher states, a tessellation is a **type** of pattern. It's made of repeating shapes that cover a surface completely without overlapping or leaving any gaps. Covering the surface completely is also called tiling the plane. Basically 3 rules for Tessellations:

18. Repeating Shapes
19. Tile the plane
20. No gaps or overlaps

A checkerboard is a tessellation made of squares. The squares meet edge to edge with no gaps and no overlapping areas. The pattern of bricks on a wall is a tessellation made of rectangles.

Over 2,200 years ago, ancient Greeks were decorating their homes with tessellations, making elaborate mosaics from tiny, square tiles. Early Persian and Islamic artists also created spectacular tessellating designs. More recently, the Dutch artist M.C. Escher used tessellations to create enchanting patterns of interlocking creatures, such as birds and fish. We will examine this more in depth tomorrow.

Making tessellations combines the creativity of an art project with the challenge of solving a puzzle. Let's look at a basic skill required to make tessellations.

Lesson on tessellations using cool math site online (pattern blocks, teacher pauses for students to experiment and make conclusions at each step)

http://www.coolmath.com/lesson-tessellations-1

(45 min.) Teacher explains what the final performance task will be and that it's skills will be practiced each day.

Performance Task: Commissioned Artist Activity explained:

A very prestigious art museum is having a contest for the cover art for their next advertising campaign that includes a brochure, posters and newspapers. This is a great opportunity for new artists like you to get exposure to increase your sales and chances for success. You are challenged to meet the requirements. The winning piece will be the one that is most representative of the theme and meets all requirements. The theme is pattern and the museum committee will be judging all entrants on the following criteria:

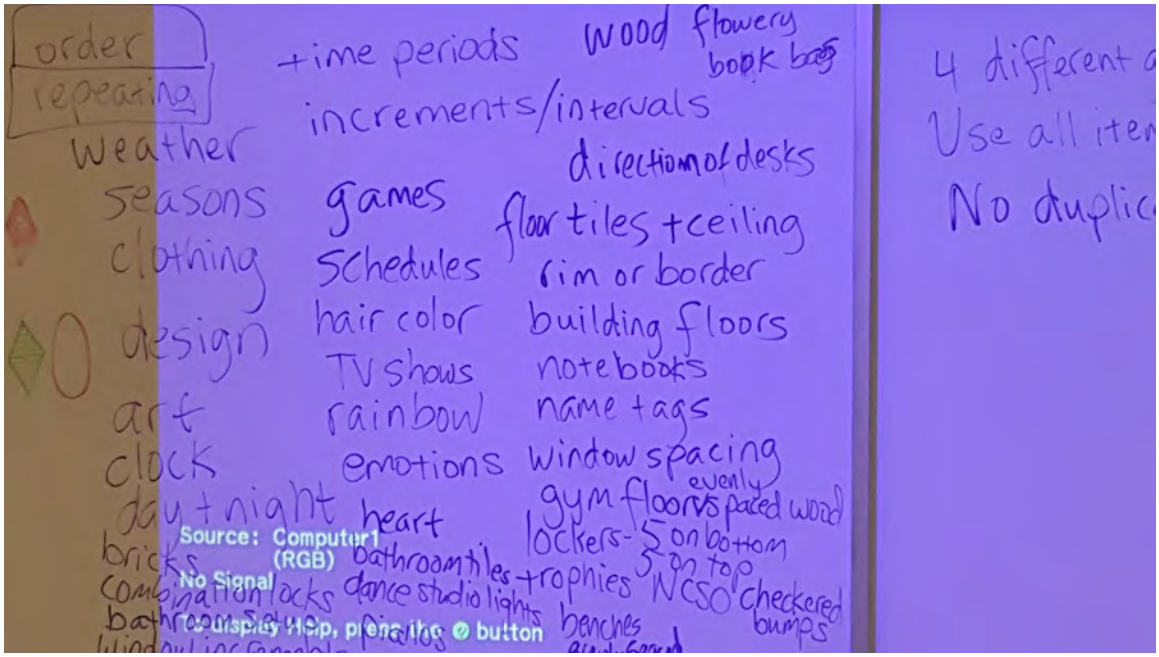
21. *Artist must include the written plan they followed to create their work of art.*
22. *Artist must make use of several types of patterns, including the three types of tessellations and at least one special pattern like Fibonacci or the Golden Ratio.*
23. *Artist must include a description of the inspiration for their art.*
24. *Artist must include a connection to nature and/or the real world.*
25. *Artist is challenged to be creative with use of pattern.*
26. *Art must be original.*

Teacher led, STUDENTS CREATE THE 1ST PHASE OF THEIR TESSELLATION, MAKING A TRANSLATION TESSELLATION (see attached)

(5 min.) clean up of materials

If early finishers, they will be taught to play the game SET which enhances pattern recognition skills. This works well in pairs or in groups.

Materials: #4 Set Card Games, clip boards, lined paper, writing utensils, expo markers, expo eraser, teacher computer, overhead, name tags, pattern blocks, Geometry Playground Exploring Tessellations packets, 3" x 5" index cards, rulers, scissors, blank paper or card stock, transparent tape, permanent colored markers/pens/pencils, tessellation examples


order
repeating
Weather
seasons
clothing
design
art
clock
day + night
bricks
combination locks
bathroom
time periods
increments/intervals
games
schedules
hair color
TV shows
rainbow
emotions
heart
bathroom tiles
dance studio lights
benches
wood
flowery
book bags
direction of desks
floor tiles + ceiling
rim or border
building floors
notebooks
name tags
window spacing
gym floors
evenly spaced words
lockers - 5 on bottom 5 on top
trophies
WCSO
checkered
bumps
4 different
Use all item
No duplic
Source: Computer1 (RGB)
No Signal
button

What are Tessellations?

Basically, a tessellation is a way to tile a floor (that goes on forever) with shapes so that there is no overlapping and no gaps. Remember the last puzzle you put together? Well, that was a tessellation! The shapes were just really weird.

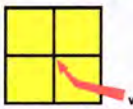
Example:

We usually add a few more rules to make things interesting!

REGULAR TESSELLATIONS:

- RULE #1: The tessellation must tile a floor (that goes on forever) with no overlapping or gaps.
- RULE #2: The tiles must be regular polygons - and all the same.
- RULE #3: Each *vertex* must look the same.

What's a vertex?



where all the "corners" meet!

What can we tessellate using these rules?

Triangles? Yep!

Notice what happens at each vertex!

The interior angle of each equilateral triangle is 60 degrees.....

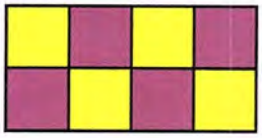
60 + 60 + 60 + 60 + 60 + 60 = 360 degrees

CONTINUE ▸

1 2 3 4

What are Tessellations?

Squares? Yep!



What happens at each vertex?

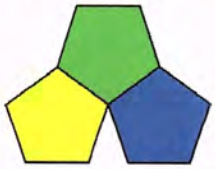
90 + 90 + 90 + 90 = 360 degrees again!

So, we need to use regular polygons that add up to 360 degrees.

Will pentagons work?

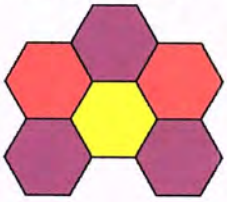
The interior angle of a pentagon is 108 degrees. . .

108 + 108 + 108 = 324 degrees . . . Nope!



Hexagons?

120 + 120 + 120 = 360 degrees Yep!



◂ PREVIOUS CONTINUE ▸

1 2 3 4

What are Tessellations?

Heptagons?

No way!! Now we are getting overlaps!

Octagons? Nope!

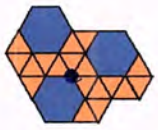
They'll overlap too. In fact, all polygons with more than six sides will overlap! So, the only regular polygons that tessellate are triangles, squares and hexagons!

SEMI-REGULAR TESSELLATIONS:

These tessellations are made by using two or more different regular polygons. The rules are still the same. Every vertex must have the exact same configuration.

Examples:

3, 6, 3, 6

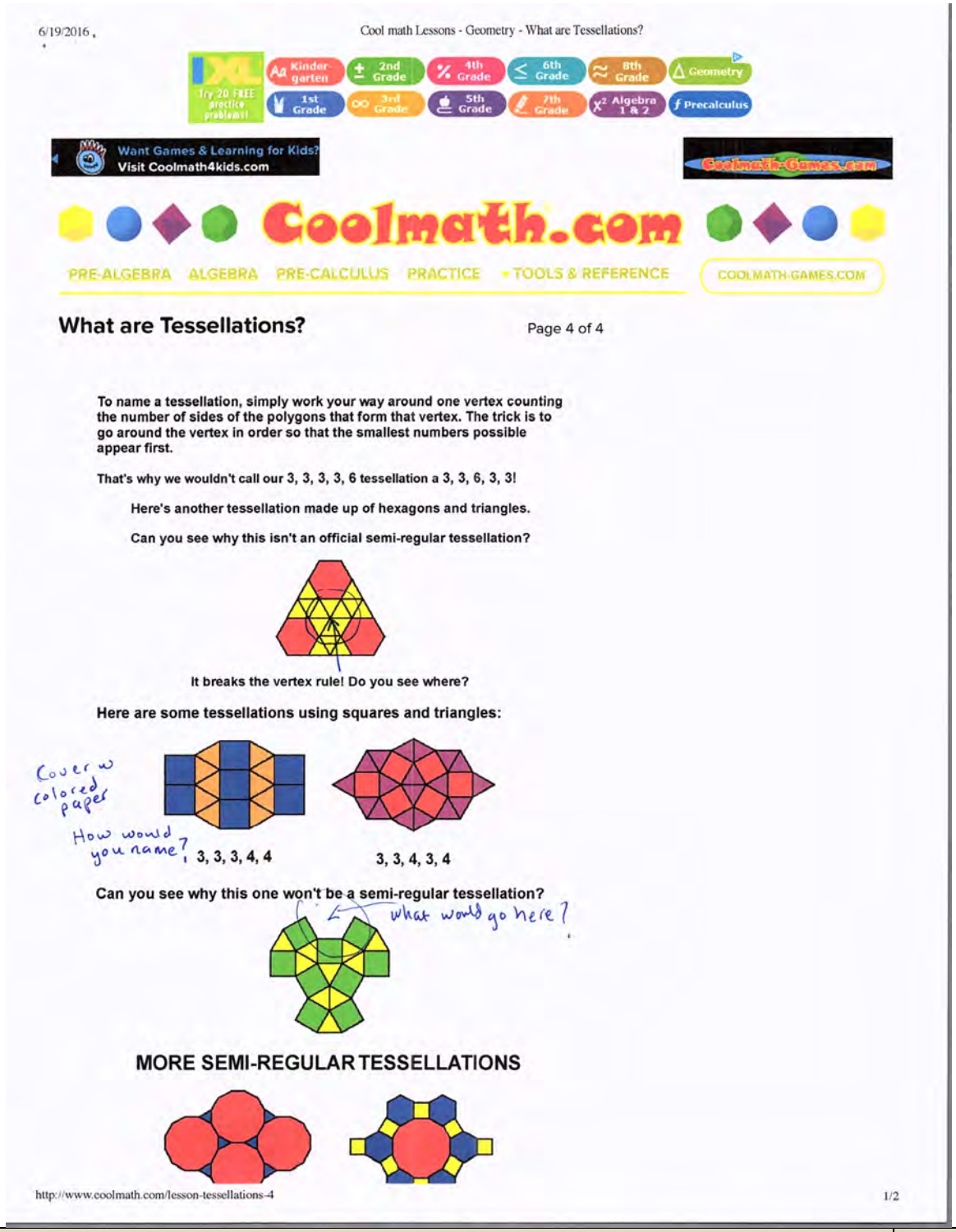


3, 3, 3, 3, 6

These tessellations are both made up of hexagons and triangles, but their vertex configuration is different. That's why we've named them!

◂ PREVIOUS CONTINUE ▸

1 2 3 4



6/19/2016 Cool math Lessons - Geometry - What are Tessellations?

Coolmath.com

PRE-ALGEBRA ALGEBRA PRE-CALCULUS PRACTICE TOOLS & REFERENCE COOLMATH-GAMES.COM

What are Tessellations?

Page 4 of 4

To name a tessellation, simply work your way around one vertex counting the number of sides of the polygons that form that vertex. The trick is to go around the vertex in order so that the smallest numbers possible appear first.

That's why we wouldn't call our 3, 3, 3, 3, 6 tessellation a 3, 3, 6, 3, 3!

Here's another tessellation made up of hexagons and triangles.

Can you see why this isn't an official semi-regular tessellation?

It breaks the vertex rule! Do you see where?

Here are some tessellations using squares and triangles:

Cover w colored paper

How would you name? 3, 3, 3, 4, 4

3, 3, 4, 3, 4

Can you see why this one won't be a semi-regular tessellation?

what would go here?

MORE SEMI-REGULAR TESSELLATIONS

http://www.coolmath.com/lesson-tessellations-4 1/2

TEACHER NAME		Lesson #
Tealecia Fletcher		2
MODEL	**CONTENT AREA**	**GRADE LEVEL**
Visual Thinking Strategy	Mathematics/ Literacy	6
CONCEPTUAL LENS	**LESSON TOPIC**	
PATTERNS	PATTERNS/TESSELLATIONS	

LEARNING OBJECTIVES *(from State/Local Curriculum)*

Common Core State Standards (CCSS)

- CCSS.MATH.CONTENT.7.G.A.2

Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or

no triangle

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- CCSS.MATH.CONTENT.8.G.A.1

  Verify experimentally the properties of rotations, reflections, and translations

- CCSS.MATH.CONTENT.8.G.A.4
  Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- CCSS.MATH.CONTENT.2.G.A.1
  Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
- CCSS.MATH.CONTENT.4.G.A.3
  Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
- CCSS.MATH.CONTENT.4.G.A.3
  Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
-

CROSS CURRICULUM

- CCSS.ELA-Literacy.RH.6-8.7: "Integrate visual information (e.g., in charts, graphs, photographs, videos or maps) with other information in print and digital texts."
- CCSS.ELA-Literacy.CCRA.SL.1: "Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively."

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	**THE ESSENTIAL QUESTION** *(What question will be asked to lead students to "uncover" the Essential Understanding)*
Patterns Inform Predictions	***How Do Artists Use Patterns To Inform Predictions?***

CONTENT KNOWLEDGE **(What factual information will students learn in this lesson?)**	**PROCESS SKILLS** **(What will students be able to do as a result of this lesson?)**
Students will know... 1. **Pattern exists in all aspects of our world.** 2. **That exploring the properties of shapes and patterns allows us to reflect and interpret our world.** 3. **Art and math have an interwoven relationship.** 4. **Vertices are where the endpoints of sides of polygons meet.** 5. **Regular Tessellations have regular polygons (same sides/same angles) and must be the same at every vertex.** 6. **That there are three regular tessellations, triangles, squares, rectangles, and hexagons.** 7. **Angles at every vertex must add up to 360 degrees.** 8. **Semi-regular tessellations have two or more regular polygons and also must be the same at every vertex.** 9. **Semi-regular tessellations may have the same polygons, but different configurations.** 10. **How to name or label tessellations using numbers.** 11. **That a prototile is a tile to which many or all other tiles in the tessellation are similar.** 12. **That regular tessellations all have a single prototile, while the semi-regular tessellations all have more than one prototile.** 13. **That all triangles tessellate, whether equilateral, isoscles, right or scalene.** 14. **That all quadrilaterals tessellate, including rectangles, parallelograms, rhombi, trapeoids and even concave quadrilaterals.** 15. **That a translation unit cell is the smallest group of tiles that can be used to tile the plane without rotation or reflection.**	**Students will be able to...** 1. Examine art from different artists and develop a personal connection to it. 2. Identify what they see in these images and to share their associations with others. 3. Enhance their language, critical and creative thinking skills from identifying objects, people and actions in art. 4. Listen to ideas of others. 5. Give examples of and explain math connections to art.

GUIDING QUESTIONS

What questions will be asked to support instruction?
Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	**During Lesson Questions:**	**Post Lesson Questions:**
1. What things did you see? 2. Did these things remind you of anything? 3. Remind me, how did we define patterns yesterday?	**While exploring the station activity:** 1. **The veins in a leaf go from larger to smaller in order to transport fluid to the leaf. This is an example of a type of pattern called a branching fractal. Can you think of other examples of branching fractals found in nature that can inform prediction?** 2. **What is symmetry?** 3. **The regular triangle tessellation above has lots of symmetry. Mark the distinct lines of mirror symmetry and distinct points of rotational symmetry (meaning mark one of each type). What relationship does symmetry have to pattern?** 4. **What part does symmetry play in patterns informing predictions?** 5. **Do you think analyzing the**	1. **What do you think interested the artist in their subject?** 2. **What elements of nature and/or pattern influenced the artist's work?** 3. **Looking at one part of a picture, were you able to predict what another part would look like?** 4. **How did patterns within the picture inform your predictions?** 5. **How did the artist use pattern to inform their predictions?**

	patterns on animals like the giraffe helps to inform predictions in some way? 6. If one scale measures 1mm across, use the number from the question above to calculate/predict the circumference of the lizard's thigh. (All the way around) Do you think the total number of scales on one rear leg of the lizard is closer to 200, 2000, 0r 20,000? Why? While looking at the art: 1. What's going on in this picture? 2. What do you see that makes you say that? 3. What else can you find? 4. What do you mean by that? Please clarify. 5. (Probing for character) Who is this person? 6. (Probing for setting) Where is this taking place? Consider color, style, and how items are represented: 7. How does it make you feel? 8. What do you like or dislike? Why do you think you feel this way? 9. What does it remind you of? 10. What is the artist's purpose? 11. Why does it matter? 12. What questions do you still have?	

DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	Learning Environment
	Visual Thinking Strategy addresses the readiness level of gifted students. Students will create the second phase of their tessellation according to their level or ability.		

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect - *This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.*

(10 min.) 12:30 – 12:40

Teacher tells the student that the video they are about to see is based on the works of a famous artist who focused on tessellations.

Inspirations Animation, A 3D animation by Cristóbal Vila based on a number of popular Escher prints*:*

http://www.mcescher.com/news/

Class discussion of video using the pre-lesson questions:

Students listen to lyrics of Math Jingle, I Repeat Patterns and restate the definition of pattern we defined yesterday:
:http://www.hbschool.com/jingles/jingles_all/35i_repeat.html

1) Remind me, how did we define pattern yesterday? Repeating units of shape or form + arranged following a rule.

2) What things did you see? Did you see anything related to pattern?

3) Did these things remind you of anything as far as pattern?

Explore - *In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.*

(50 min.)* (Includes 5 min. end break) *12:40 – 1:30

Teacher directed exploration of 8 stations (40 min.) (PATTERN HISTORY/GEOGRAPHY/PREDICTIONS):
Students will rotate through 8 stations, which are large tri-folds. Each tri-fold has a topic depicted and a written description that elaborates on the rules students learned about tessellations on Monday, and has them explore a design(s) for how it relates to pattern, a geographical location and /or history, and the world around them.

1) Introduction to Edge-to-Edge Tessellations and sums of interior angles.
   a) Islamic architecture in the Middle East
2) Symmetry in patterns, especially tessellations
3) Real World Tessellations
4) Africa (Eastern, Middle, Northern, Southern, Western)
5) America (Latin America and the Caribbean, South America, Caribbean, Central America, Northern America)
6) Asia (Central, Eastern, Southern, South-Eastern, Western)
7) Europe (Eastern, Northern, Southern, Western)
8) Oceania (Australia and New Zealand, Melanesia, Micronesia, Polynesia)

(10 min.) Class summarizes/makes observations about each station activity. The teacher asks the during the lesson questions that were at each station and each group adds any other comments not already shared. The teacher poses additional questions for each station:

1. **The veins in a leaf go from larger to smaller in order to transport fluid to the leaf. This is an example of a type of pattern called a branching fractal. Can you think of other examples of branching fractals found in nature that can inform prediction?**
2. **The regular triangle tessellation above has lots of symmetry. Mark the distinct lines of mirror symmetry and distinct points of rotational symmetry (meaning mark one of each type). What relationship does symmetry have to pattern?**
3. **What part does symmetry play in patterns informing predictions?**
4. **Do you think analyzing the patterns on animals like the giraffe helps to inform predictions in some way?**
5. **If one scale measures 1mm across, use the number from the question above to calculate/predict the circumference of the lizard's thigh. (All the way around) Do you think the total number of scales on one rear leg of the lizard is closer to 200, 2000, 0r 20,000? Why?**

Teacher states you are about to view the art of the most famous pattern/tessellation artist, for our next activity.

(40 min.) 1:30– 2:10 VISUAL THINKING STRATEGY LESSON (30min.)

Strategically picked tessellation art is posted on the overhead.

M.C. Escher online Gallery
http://www.mcescher.com/
Bond of Union

http://www.mcescher.com/gallery/recognition-success/bond-of-union/

TEACHER EXPLAINS RULES FOR SHARING:

1. *Must raise your hand and be called on to share.*
2. *Must listen with focus and patience.*
3. *You are allowed to disagree, but be polite not mean.*

Teacher asks; as you look at the art, think…
Teacher gives plenty of wait time, to allow students to think critically.

1. **What's going on in this picture?**
2. **What do you see that makes you say that?**
3. **What else can you find?**
4. **What do you mean by that? Please clarify.**
5. **(Probing for character) Who is this person?**
6. **(Probing for setting) Where is this taking place?**

Consider color, style, and how items are represented:

7. **How does it make you feel?**
8. **What do you like or dislike? Why do you think you feel this way?**
9. **What does it remind you of?**
10. **What is the artist's purpose?**
11. **Why does it matter?**
12. **What questions do you still have?**

Thank you for sharing, I greatly enjoyed your visual thinking and sharing.

Explain - *Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.*

(10:30) Students watch portion of the video on the artist*: Interview of George Escher, son of M.C. Escher on what it was like with his father.*
http://www.mcescher.com/about/interview-with-eschers-son/
And teacher asks:

1. **How do you think pattern influenced Escher's art and his choices?**
2. **Does knowing more about the artist change any of your opinions?**
3. **Does Escher use pattern to inform predictions?**

Elaborate —*Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways*

(45 min.) 2:10 – 2:55 COMPUTER STATION EXPLORATION OF OTHER ARTISTS

Students spend 5 minutes at each station and rotate through other pattern artists. Students are directed to focus on (the post lesson questions):

1. **What do you think interested the artist in their subject?**
2. **What elements of nature and/or pattern influenced the artist's work?**
3. **Looking at one part of a picture, were you able to predict what another part would look like?**
4. **How did patterns within the picture inform your predictions?**
5. **How did the artist use pattern to inform their predictions?**

Andrew Crompton
http://www.cromp.com/pages/tess1.html

Wolter Schraa fractal art
https://wolter.home.xs4all.nl/index.html

David E. Joyce
http://aleph0.clarku.edu/~djoyce/poincare/poincare.html

Bruce Bilney
http://www.ozzigami.com.au/tessellations.html

Roger Penrose (Penrose triangle and stairs)
http://www.break.com/article/how-the-rit-stairwell-illusion-works-2440836

Evaluate: *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*

(45 min.) 2:55 – 3:40 Class discussion using the post lessons questions the teacher provided. Groups share their thoughts and are reminded they are answering specifically related to pattern. (10 min.)

1. **What do you think interested the artist in their subject?**
2. **What elements of nature and/or pattern influenced the artist's work?**
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STUDENTS CREATE THE 2ND PHASE OF THEIR TESSELLATION EVALUATION, REFLECTING A TESSELLATION (see attached) (35 min.)

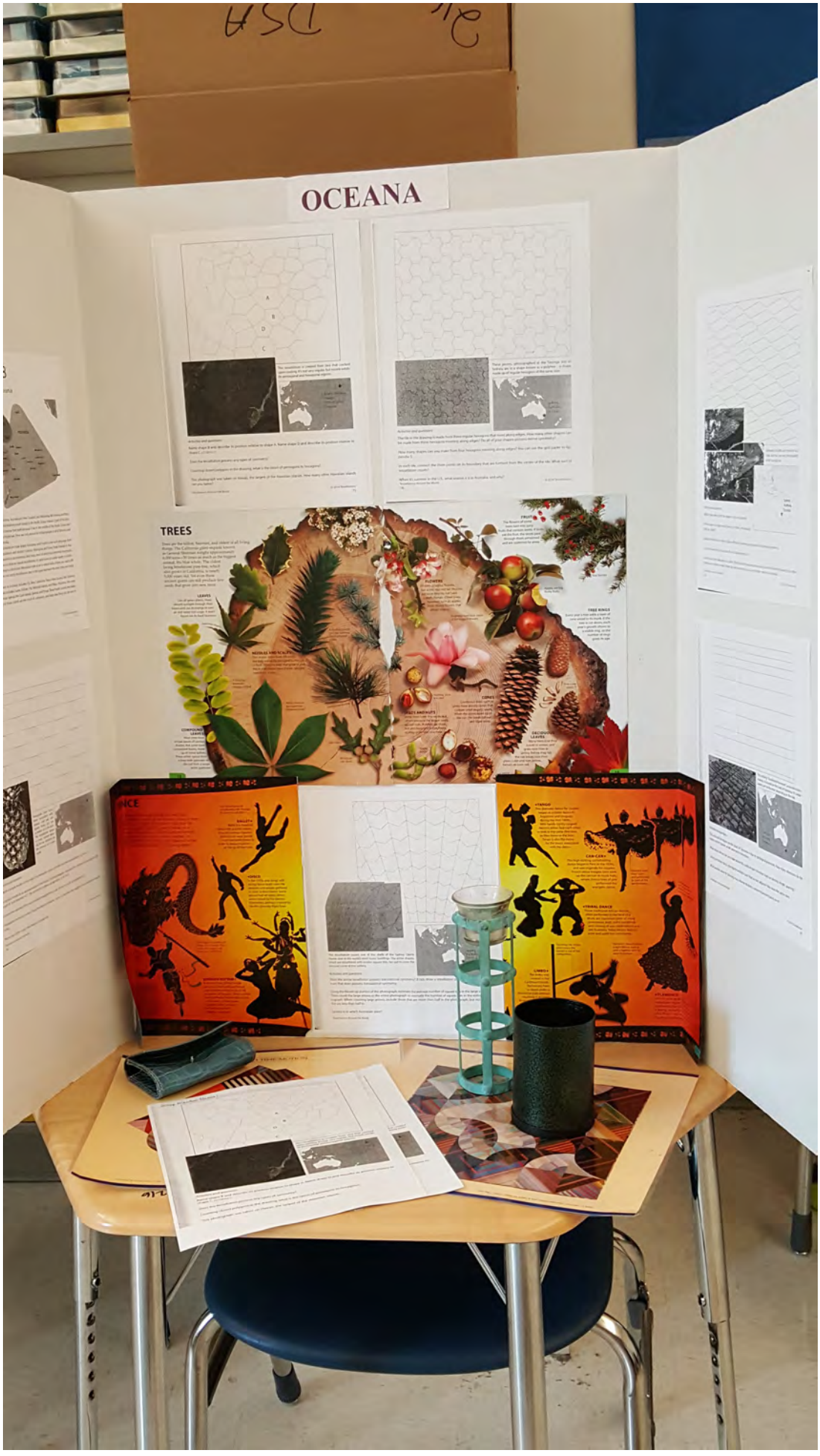
(5 min. clean up of materials) Students line up

IF EARLY FINISHERS, THEY WILL PLAY AN ONLINE GAME EXPLORING NUMBER PATTERNS

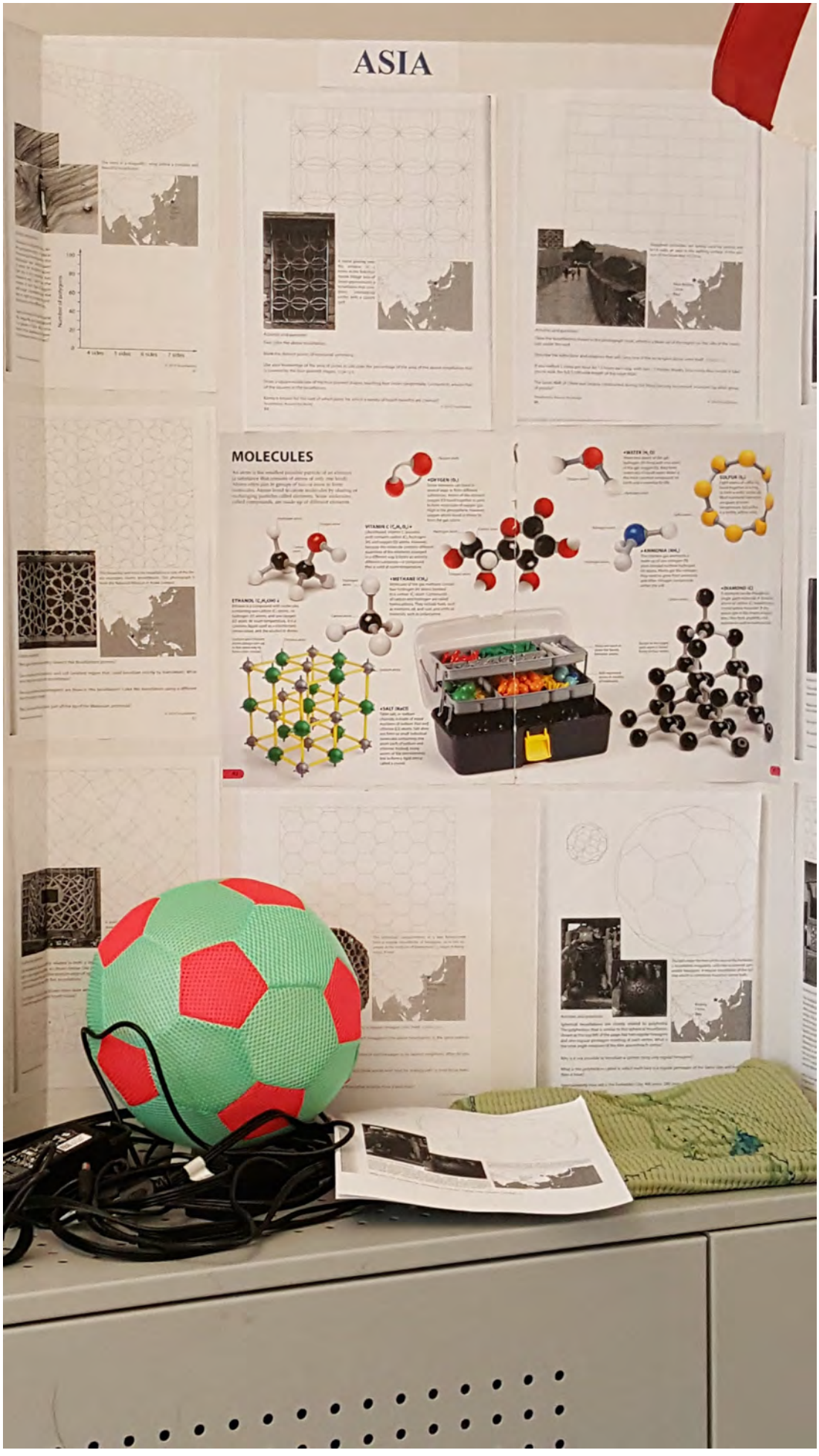
Number Cracker Game:
http://www.funbrain.com/cracker/index.html

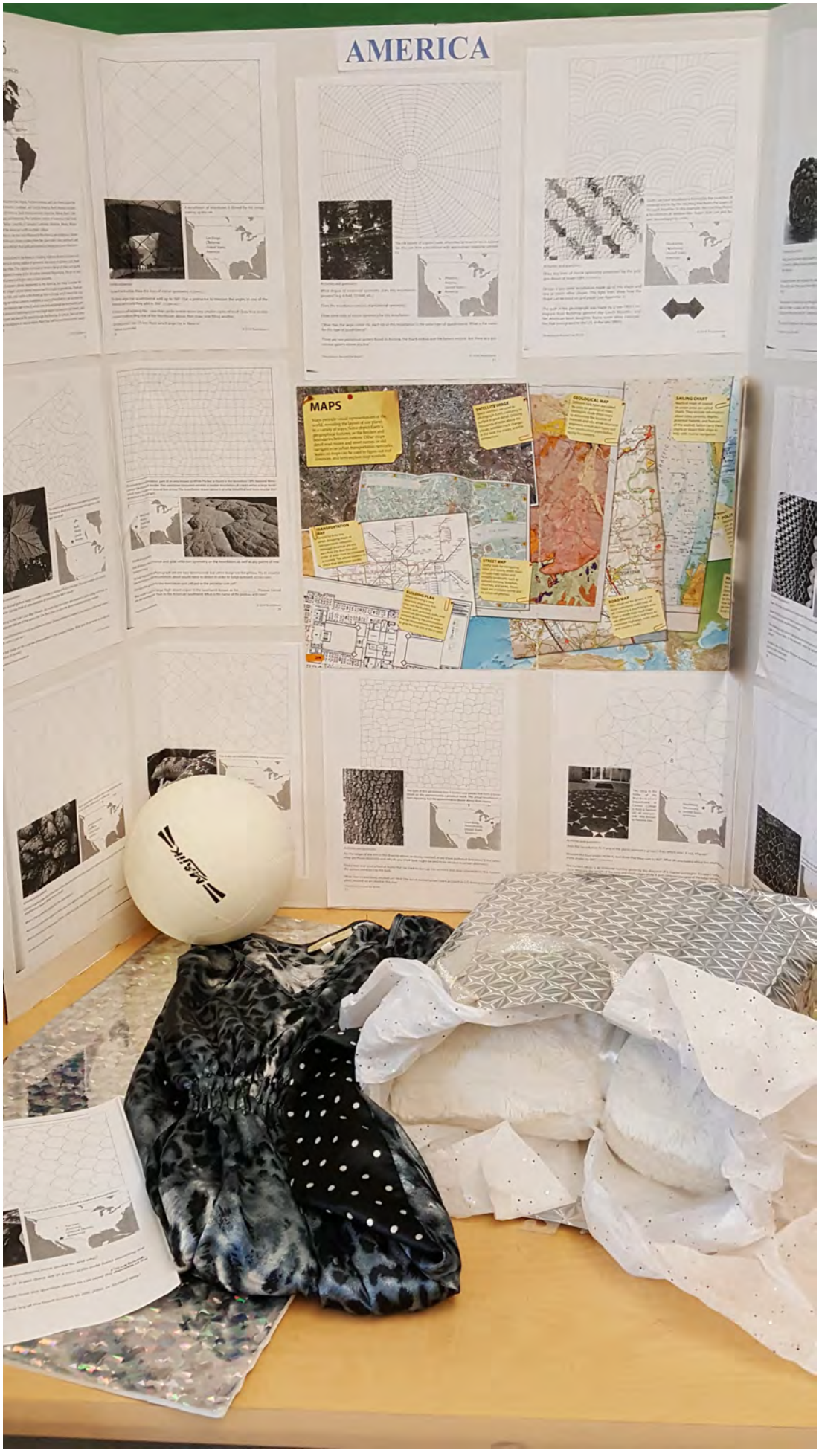

TESSELLATION
INTRODUCTION
SYMMETRY


AFRICA
4
REPTILES
WINDOW PIECE
COUNTERPOINT


OCEANA
TREES


3
Real-World Tessellations
COMPOSITION 2


ASIA
MOLECULES


AMERICA
MAPS
SATELLITE IMAGE
GEOLOGICAL MAP
SAILING CHART
TRANSPORTATION MAP
STREET MAP
BUILDING PLAN
ROAD MAP


EUROPE
ESTONIAN AIR
GENETICS
DNA

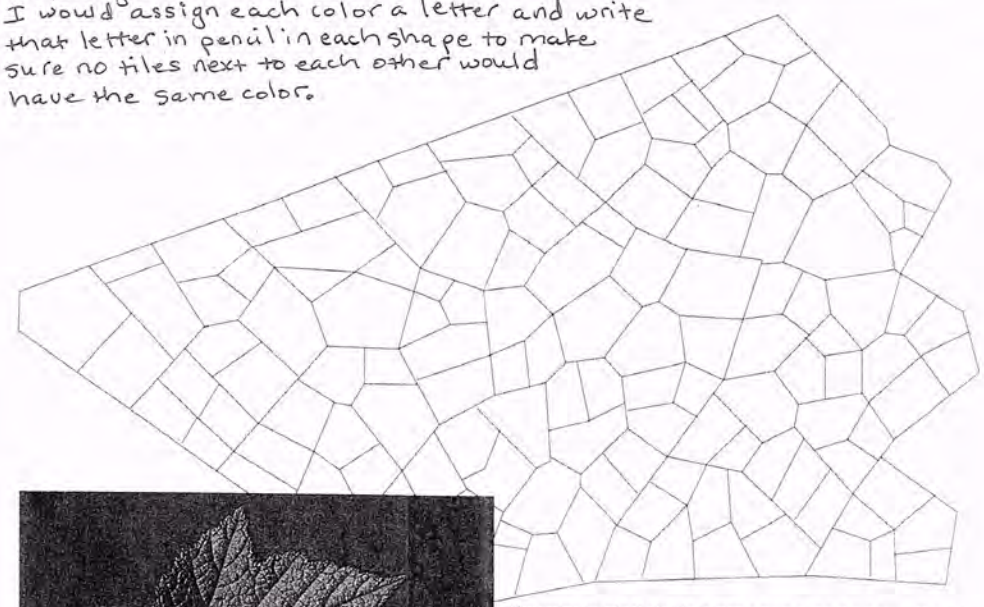

Algeblocks
SYMMETRY
SHELLS

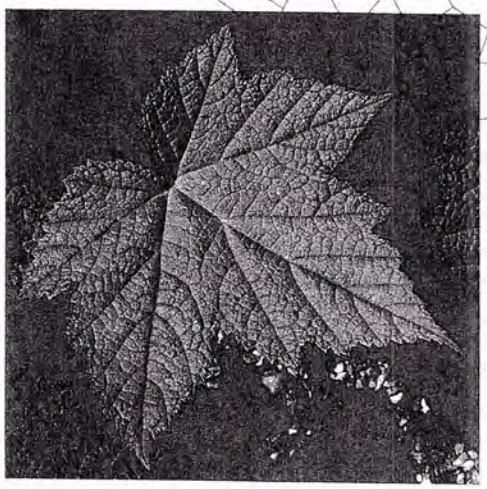

LIFE CYCLES

For the following tri-fold partner/group activity, I have attached the student and teacher versions. Teacher versions have clarifying details.

Group Member Names:

* Before you color → Plan! I would assign each color a letter and write that letter in pencil in each shape to make sure no tiles next to each other would have the same color.





The veins in a leaf divide it into a non-repeating tessellation. The drawing above is an approximation of a portion of the leaf show at left.

Activities and questions:

The veins in a leaf go from larger to smaller in order to transport fluid to the leaf. This is an example of a branching fractal. Can you think of other examples of branching fractals in nature?

According to the Four Color Map Theorem, no more than four colors are needed to color a tiling such that no two adjacent tiles are of the same color. Use four colors to color the above tessellation in this manner.

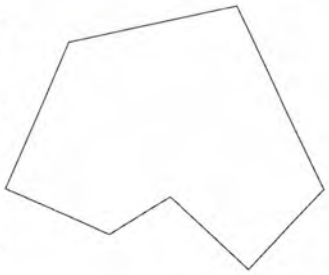
Masks plan an important role in many traditional African ceremonies. This warthog mask employs a variety of geometric markings, including a regular tessellation of triangles in the narrowest portion of the mask.

Activities and questions:

The regular triangle tessellation above has lots of symmetry. Mark the distinct lines of mirror symmetry and distinct points of rotational symmetry (i.e., mark one of each type).

Color the tessellation above with two colors such that no two adjacent triangles have the same color. What do you notice about the triangles of each color?

The mask shown above uses triangles to cover much of its surface. Similarly, a mesh of triangles is commonly used in computer graphics to model three-dimensional objects. Note that any polygon in the plane can be broken into triangles. Divide the polygon at right into triangles two different ways. What is the minimum number of triangles required?



Use a book on African masks or the internet to find some other motifs commonly used in them.

Group Member Names:

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Activities and questions:

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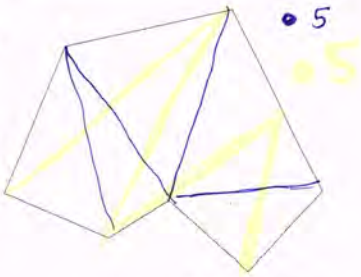
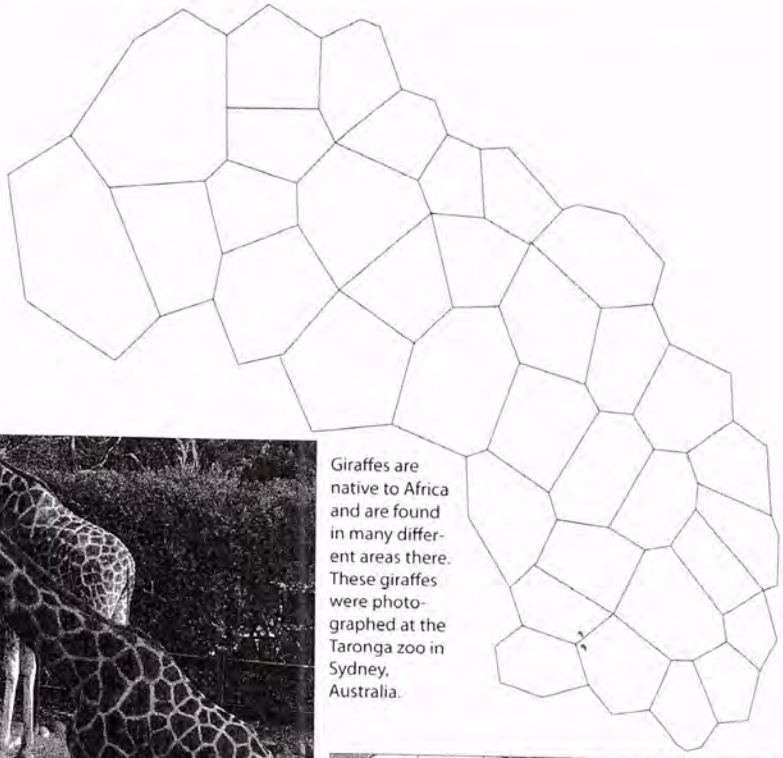
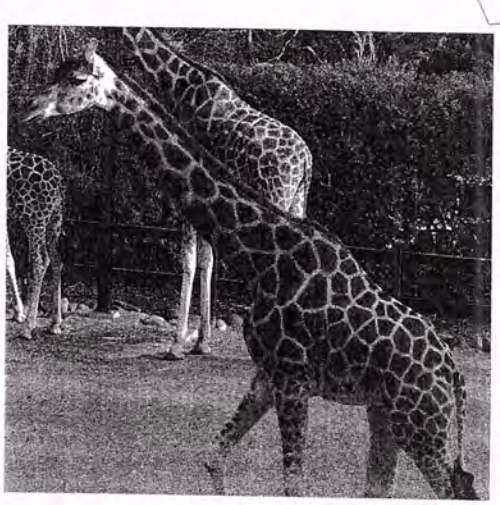


How were △s used to help inform predictions?
— comp graphics, modeling 3D

Group Member Names:





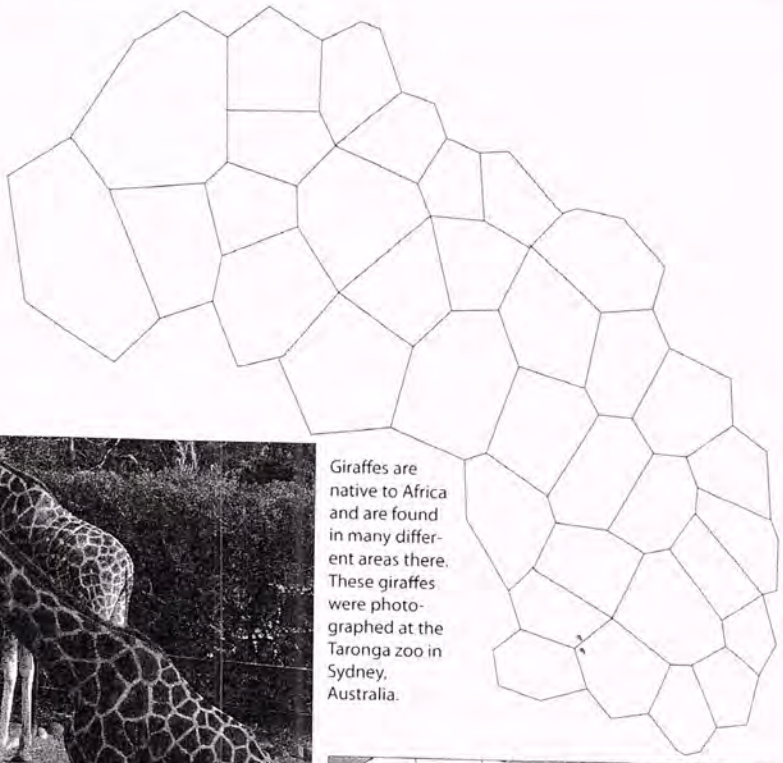
Giraffes are native to Africa and are found in many different areas there. These giraffes were photographed at the Taronga zoo in Sydney, Australia.

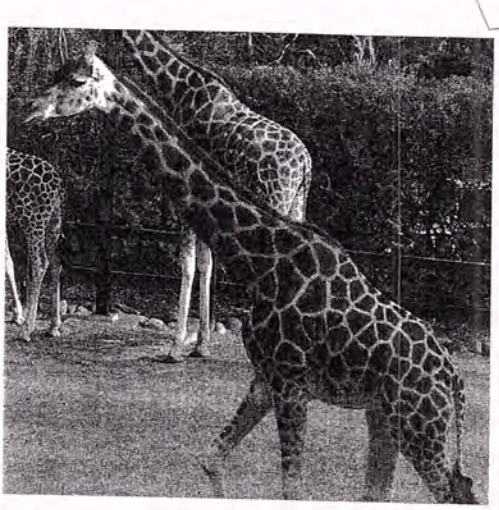
Activities and questions:

A giraffe's markings can be approximated as polygons. Describe the markings on the side of the giraffe as quantitatively as possible. Approximately how many polygons are needed to cover one side? How many sides do they have (minimum to maximum number and most common number)? *(CCSSM G-MG 1)*

Make a histogram showing the number of polygons vs. the number of sides in the polygon from the drawing above.

Group Member Names:





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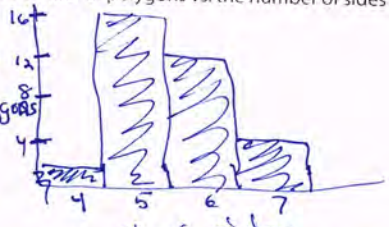
using paths / counting / size

36

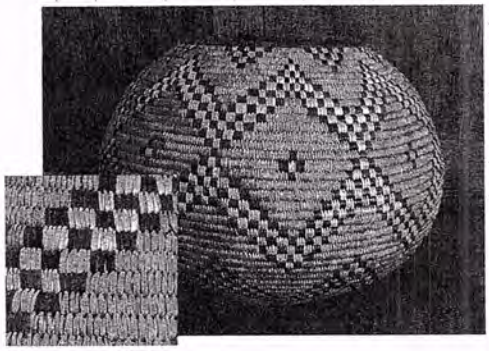
Make a histogram showing the number of polygons vs. the number of sides in the polygon from the drawing above.

4 | (1)
5 HH HH HH | (16)
6 HH HH || (12)
7 HH (5)



How do you think counting the polygons or noting the pattern of the shapes helps inform predictions? — Identification / tracking / wildlife conservation

Group Member Names:



This African basket, purchased in the US, uses a zigzag design made up of small squares arranged in one of the three regular tessellations.

Activities and questions:

The regular square tessellation drawn above is perhaps the simplest and most common of tessellations. Name some examples of this tessellation in your world.

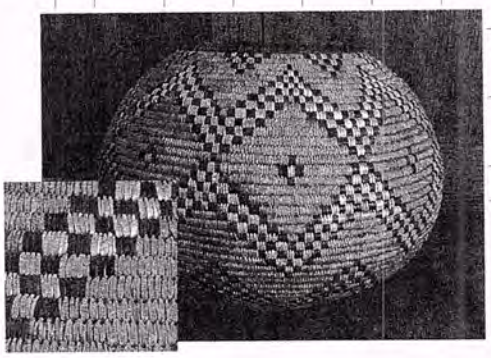
If the square tessellation were stretched out in the horizontal direction, what would the new shape of the tiles be?

If the square tessellation were stretched out along a line at 45 degrees to the horizontal , what would the new shape of the tiles be?

The three tile shapes described above are all quadrilaterals. In the space at right, draw some quadrilaterals that are not squares, rectangles, or rhombuses. *(CCSSM 3.G.1)*

Basket weaving is an activity that has been practiced in cultures around the world for thousands of years. What types of things do you think primitive cultures would use baskets for?

Group Member Names:



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Activities and questions:

The regular square tessellation drawn above is perhaps the simplest and most common of tessellations. Name some examples of this tessellation in your world.
checkerboard, floor tiles, ceiling tiles

If the square tessellation were stretched out in the horizontal direction, what would the new shape of the tiles be? rectangles

If the square tessellation were stretched out along a line at 45 degrees to the horizontal , what would the new shape of the tiles be? parallelograms

The three tile shapes described above are all quadrilaterals. In the space at right, draw some quadrilaterals that are not squares, rectangles, or rhombuses. *(CCSSM 3.G.1)*

trapezoid

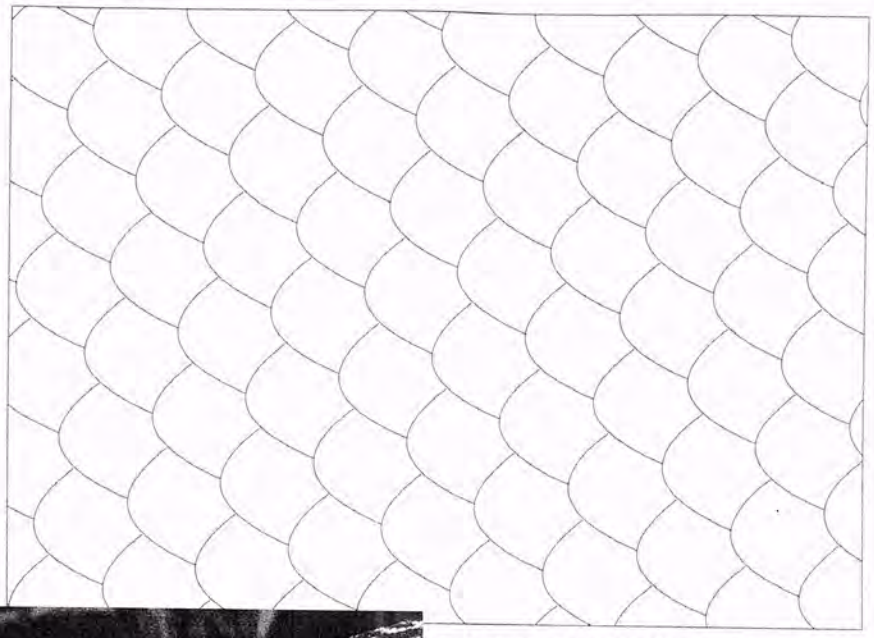
kite

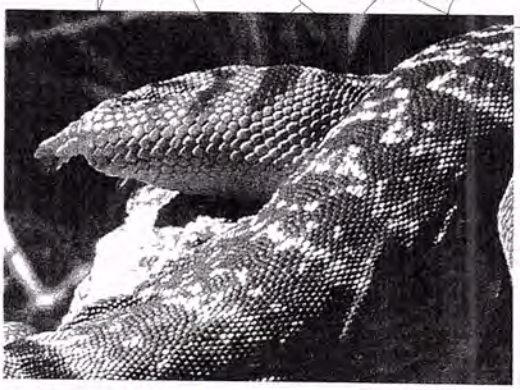
Basket weaving is an activity that has been practiced in cultures around the world for thousands of years. What types of things do you think primitive cultures would use baskets for? storage / carrying

why would the basket maker need to make predictions?

— size of the basket
shape
uniformity

Group Member Names: _____





The scales on this lizard form a natural tessellation.

Activities and questions:

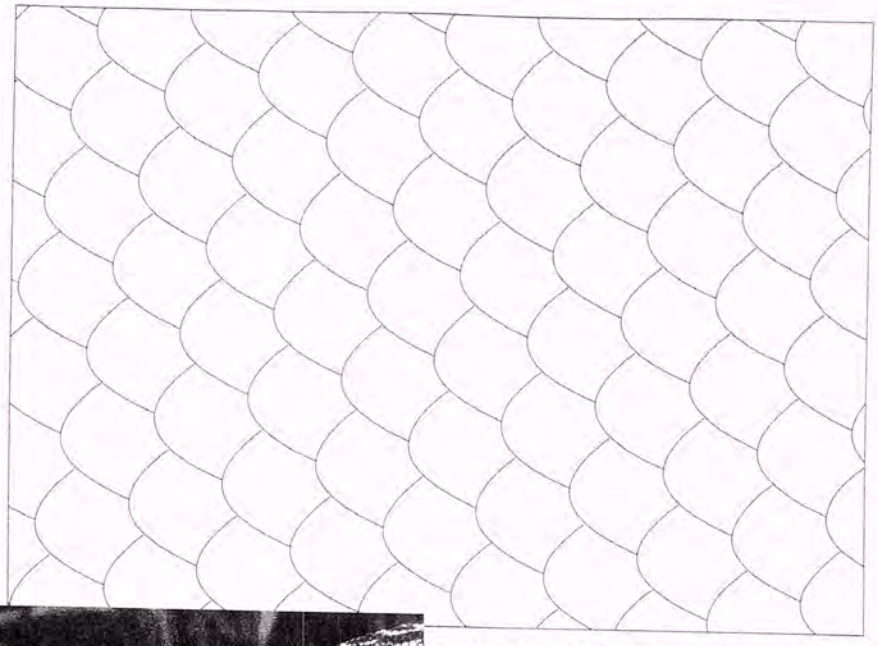
Which of the three regular tessellations is the above tessellation most similar to, and why?

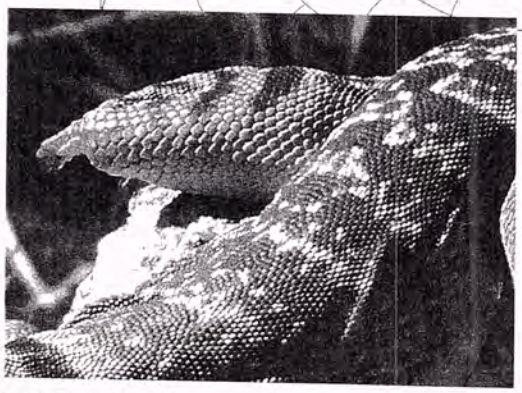
Looking at the photograph, estimate the number of scales there are in a one-scale-wide band encircling the lizard's thigh.

If one scale measures 1 mm across, use the number from the question above to calculate the ~~diameter~~ circumference of the lizard's thigh. (All the way around)

Do you think the total number of scales on one rear leg of the lizard is closer to 200, 2000, or 20,000? Why?

Group Member Names:





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Activities and questions:

Which of the three regular tessellations is the above tessellation most similar to, and why?

– quadrilateral (rectangle or square)

Looking at the photograph, estimate the number of scales there are in a one-scale-wide band encircling the lizard's thigh. 56 row of 14 middle to the top + 14x4=56

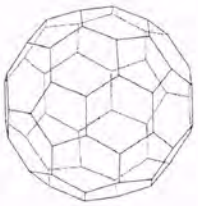
If one scale measures 1 mm across, use the number from the question above to calculate the ~~diameter~~ circumference of the lizard's thigh. (All the way around) 56x1mm = 56mm

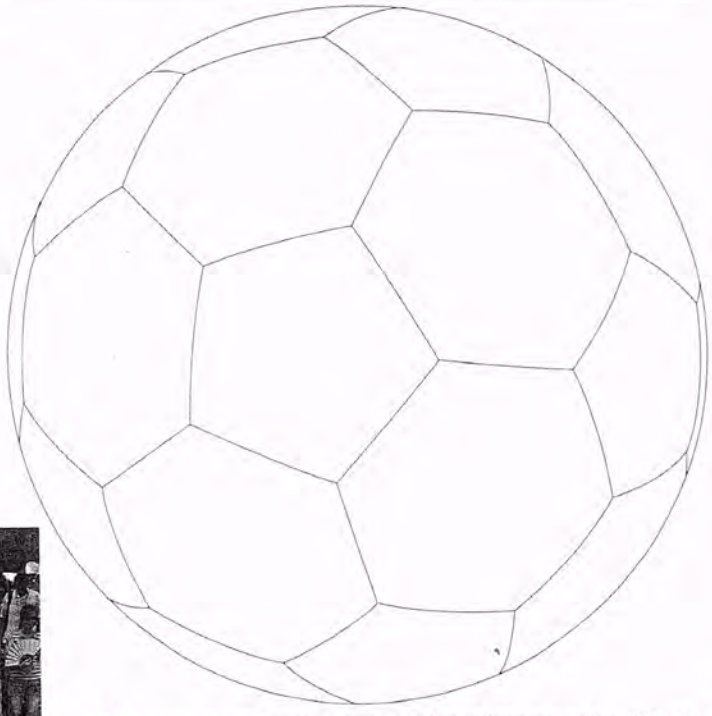
Do you think the total number of scales on one rear leg of the lizard is closer to 200, 2000, or 20,000? Why?

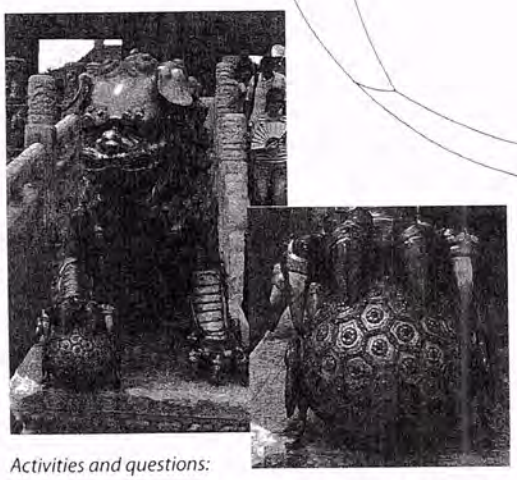
length ~13 — 2000

56
x13
168
560
728

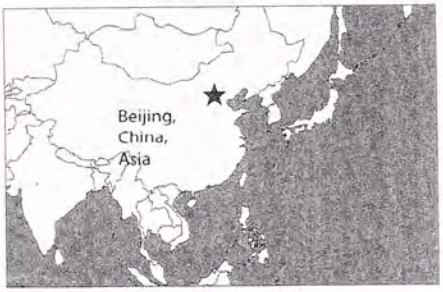
Group Member Names:







The ball under the foot of this lion in the Forbidden City is tessellated irregularly, with the occasional pentagon amidst hexagons. A regular tessellation of the sphere is that which is commonly found on soccer balls.

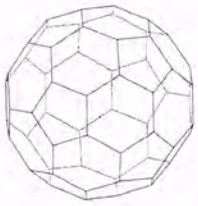


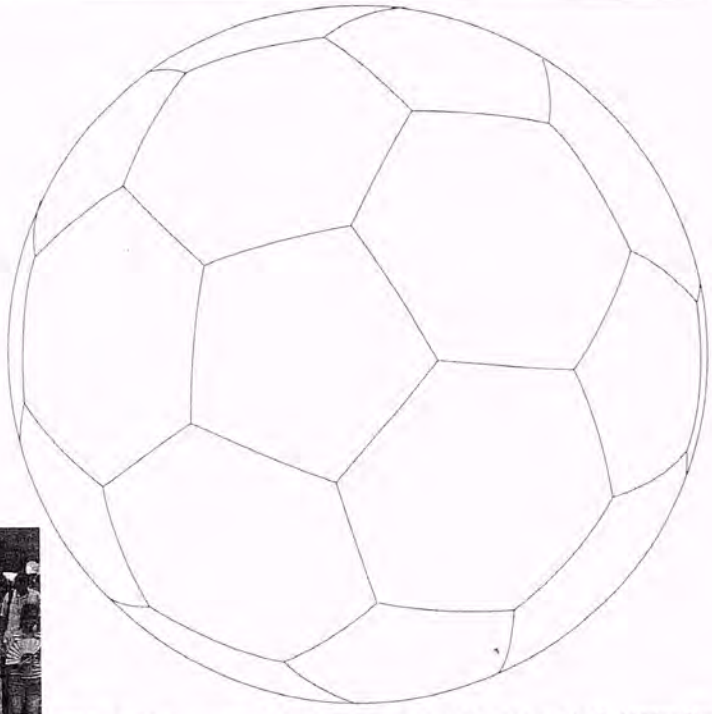
Activities and questions:

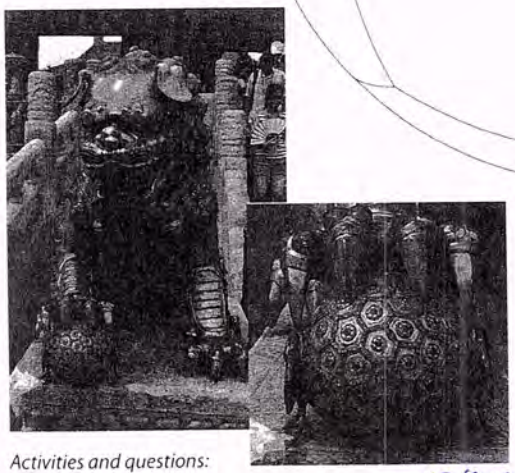
Spherical tessellations are closely related to polyhedra. The polyhedron that is similar to this spherical tessellation, shown at the top left of the page, has two regular hexagons and one regular pentagon meeting at each vertex. What is the total angle measure of the tiles around each vertex?

Why is it not possible to tessellate a sphere using only regular hexagons?

Group Member Names:







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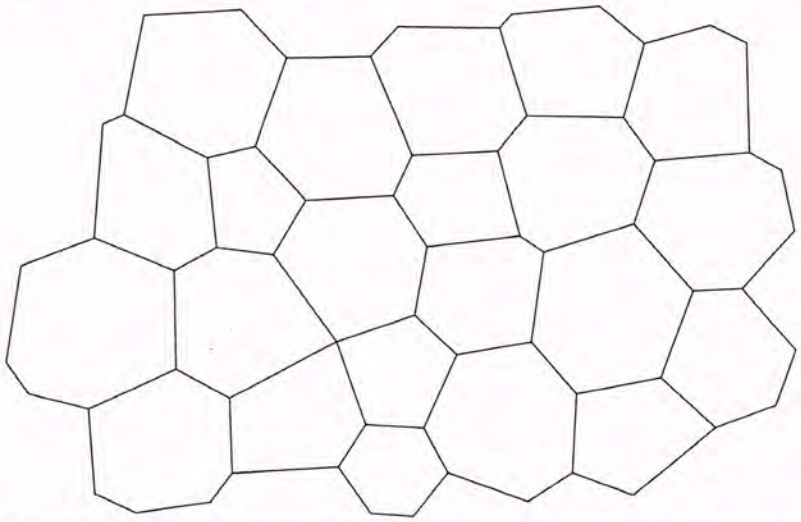
3D figures

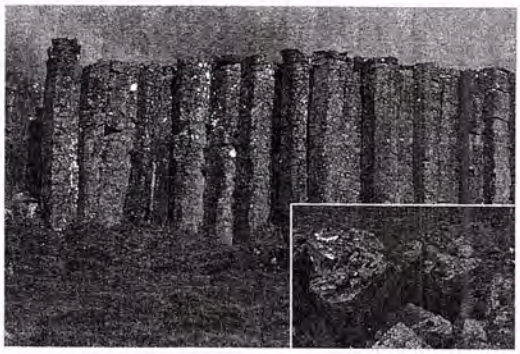
360°

Why is it not possible to tessellate a sphere using only regular hexagons?

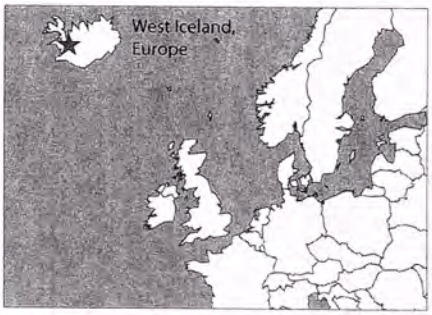
would have gaps / overlaps

Group Member Names: _____



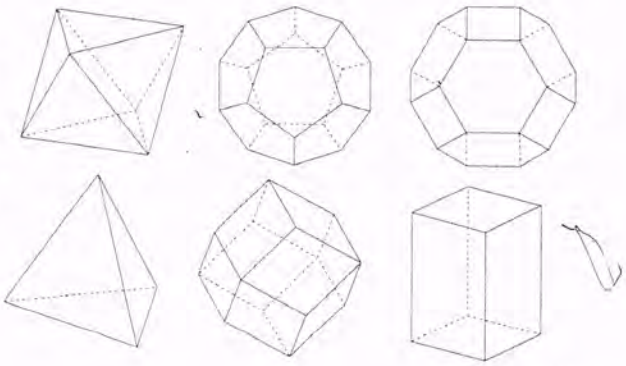


Basalt columns such as these are often described as hexagonal, but they really exhibit polygons with different numbers of sides.

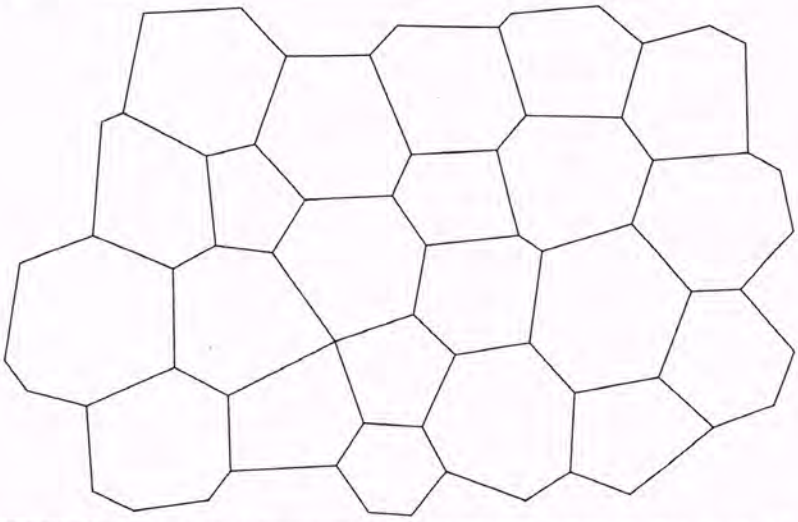


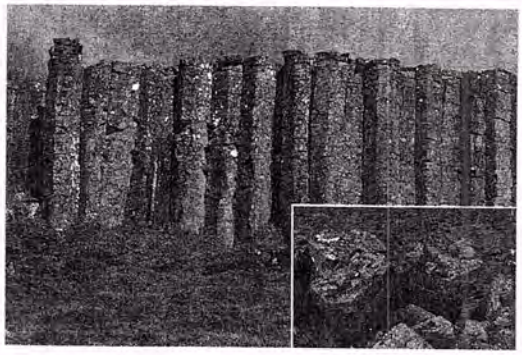
Activities and questions:

The hexagonal basalt columns approximate hexagonal prisms. Hexagonal prisms that are all the same size will tessellate three-dimensional space; i.e., they can fill space without gaps or overlaps. There are other polyhedra that have this property, though most of them don't. Circle the polyhedra at right that you think will tessellate space.



Group Member Names: _____





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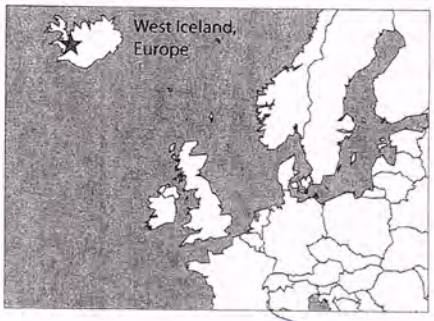

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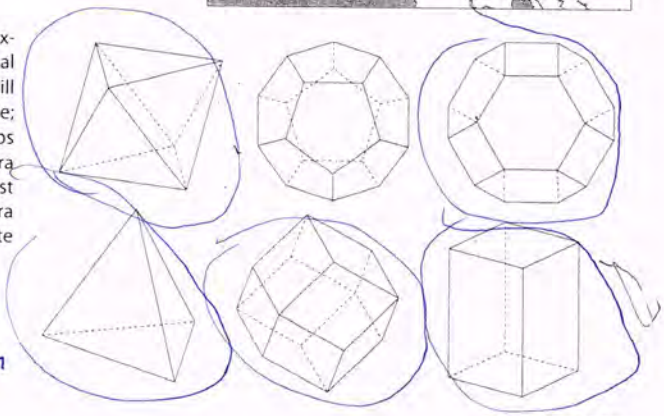
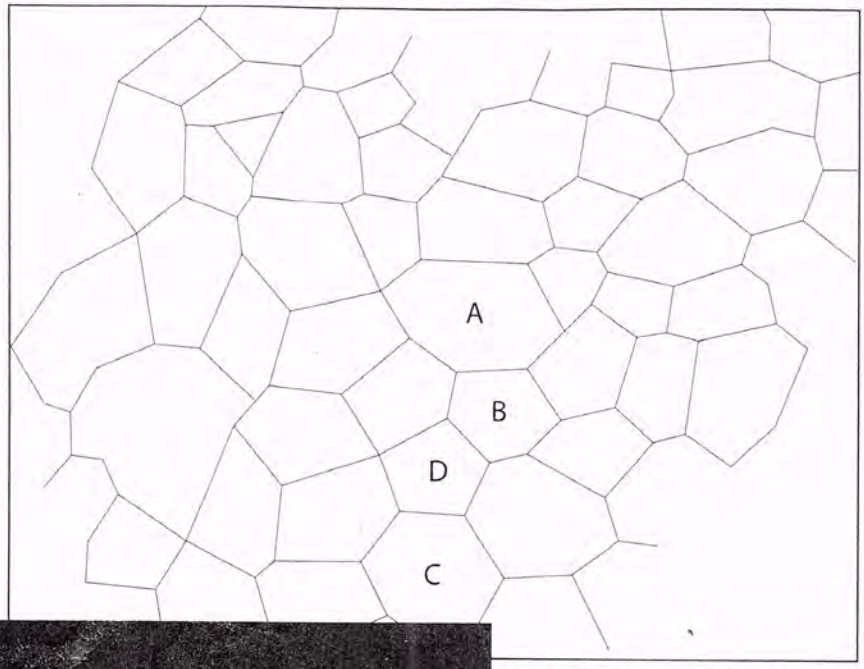


– made of reg tess items
△, square, rectangle
parallelogram, hexagon

Group Member Names: _____



This tessellation is created from lava that cracked upon cooling. It's not very regular, but mostly exhibits pentagonal and hexagonal regions.

Activities and questions:

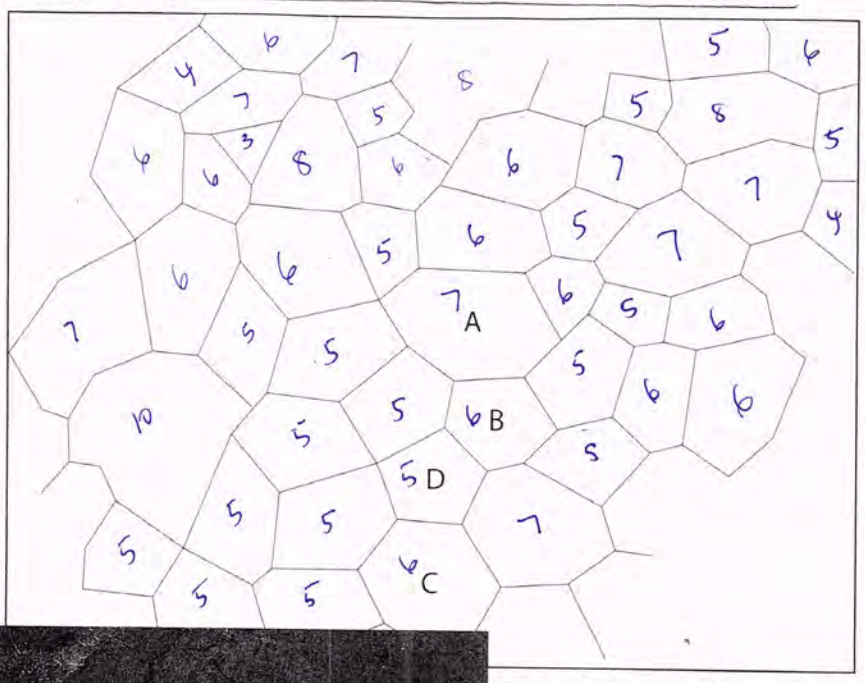
Name shape B and describe its position relative to shape A. Name shape D and describe its position relative to shape C. *(CCSSM K.G 1)*

Does the tessellation possess any types of symmetry?

Counting closed polygons in the drawing, what is the ration of pentagons to hexagons?.

This photograph was taken on Hawaii, the largest of the Hawaiian islands.

Group Member Names:



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Activities and questions:

Name shape B and describe its position relative to shape A. Name shape D and describe its position relative to shape C. (CCSSM K.G.) (B) hexagon/below (D) petagon/above

Does the tessellation possess any types of symmetry? why?
no, shapes are irregular

Counting closed polygons in the drawing, what is the ratio of pentagons to hexagons?. $\approx \frac{20}{16}$ or $\frac{5}{4}$

This photograph was taken on Hawaii, the largest of the Hawaiian islands.
What do you think the crack patterns could tell you about the flow of lava?
Why would anyone want to predict lava flow?
How do patterns inform predictions with respect to lava flow?
(cool fast → a lot more cracks → smaller pieces

Discussion on Bond of Union Lithograph Painting

Length: 370 words (1.1 double-spaced pages)
Rating: Red (FREE)

Discussion on Bond of Union Lithograph Painting

M.C. Escher was born in June 1898 to March 1972. He was a Dutch Graphic Artist most recognized for his repeating geometric patterns (tessellations) and incredible technique for illusions. He was studied and greatly appreciated by respected mathematicians and scientists yet he had no formal training in math or science. All of his work requires more than just a quick glance as you never know what you might miss the first time around. He is one of America's top selling artists. M.C. Escher's talent is obvious and unquestioned. In M.C. Escher's Bond of Union artwork, it not only tells a story, they pull the viewer into the emotional life of M.C. Escher creation.

The painting is called 'Bond of Union' by M. C Escher. The painting was set in the surrealism movement.

The painting is of two people, but is distorted as if the face was a orange peel. It doesn't seem ralistic but more futuristic and surreal, because the faces seem simple, but more robotic in some way and the floating spheres make it seem more futuristic. The painting could tell a story, But it makes the viewer really think as to what they are looking at, as the paintings meaning is not straight forward and could confuse the viewer. The painting is more about the shapes and patterns it forms, rather than the colour as the painting is only black a white and doesn't use colour to express its emotions. It could be something frpm imagination or a type of illusion. It is hard to define what kind of mood it gives, because theres no colour to the painting and the faces of the two people, doesn't really show any expression, they just seem neutral, but more happy than sad. The painting itself, to me, seems very unusual and bizarre as the the painting doesn't show a background or layout of where the painting is set in, but the objects of the painting seem to be floating in mid-air somwhere. The painting

is detailed mainly on the facial features and the tonal balance, but doesn't show any detail such as pattern. It doesn't give a general impression as you have to think about what it could mean. The painting is very tidy with neat lines and the shapes are very bold. The painting is very busy because there are a lot of the floating objects surrounding the two heads. The two heads are shown as if they were like orange peels, but joining at the top and bottom to make one big orange peel. All the spheres are evenly proportion, all the same size in the background and foreground, so they look as if they vary in size. The shapes that are used are not unusual in anyway, but are basic shapes, but still makes the painting look unrealistic and the observer confused. The lines shown in the painting are very crisp, clean cut lines but not so musch hard edged.

TEACHER NAME	Lesson #
Fletcher	3

MODEL	CONTENT AREA	GRADE LEVEL
Bruner	Mathematics	6

CONCEPTUAL LENS	LESSON TOPIC
Patterns	Tessellations (Artists use specific tools and methods, including tessellations.)

LEARNING OBJECTIVES *(from State/Local Curriculum)*

CCSS.MATH.CONTENT.K.G.1
DESCRIBE OBJECTS IN THE ENVIRONMENT USING NAMES OF SHAPES, AND DESCRIBE THE RELATIVE POSITION OF THESE OBJECTS.

CCSS.MATH.CONTENT.2.G.A.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles,

quadrilaterals, pentagons, and hexagons.

CCSS.MATH.CONTENT.4.G.A.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

CCSS.MATH.CONTENT.4.G.A.3

Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into

matching parts. Identify line-symmetric figures and draw lines of symmetry

G-CO Experiment with transformations in the plane.

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-GMD Visualize relationships between two-dimensional and three-dimensional objects.

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects.

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	THE ESSENTIAL QUESTION *(What question will be asked to lead students to "uncover" the Essential Understanding)*
PATTERNS INFORM PREDICTIONS	***How do Patterns Inform Predictions?***

CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	PROCESS SKILLS (What will students be able to do as a result of this lesson?)
Students will know… **1. That artists use specific tools and methods to inform prediction.** **2. That artists are inspired and influenced by pattern.** **3. That artists design what they make according to the rules of patterns.** **4. That artists pose and answer questions as they create, build understandings and draw conclusions in a variety of different ways.**	**Students will be able to…** **1. manipulate triangles, squares, hexagons, trapezoids and rhombuses to learn the names of the polygons, associate the names with the correct polygons, and to see how they might combine to form patterns in the plane.** **2. Reason with shapes and their attributes.** **3. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of sides. (Note: Sizes are compared directly or visually, not compared by measuring.)** **4. Listen and respond to directions using the accurate names and characteristics of the polygons.** **5. Discover which of these patterns are tessellations and which are not using the definition that a tessellation is a tiling with shapes that cover the plane without gaps or overlaps.** **6. Define and apply the terms: tessellation, polygon, equilateral, equiangular, reflection, rotation, translation, regular, semi-regular, angle, plane, vertex, tiling and adjacent.** **7. Determine which patterns are *regular* tessellations and which are not using the definition that a *regular* tessellation is a tiling with shapes that cover the**

	plane in a regularly repeating pattern without gaps or overlaps.

GUIDING QUESTIONS

What questions will be asked to support instruction?
Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:

1. **What do artists do?**
2. **What do artists wear?**
3. **What are some of the tools that an artist uses?**
4. **Where might artists work? Describe these spaces.**
5. **What characteristics should artists possess?**
6. **Are there rules artists must follow that impact their work?**
7. **What methods do artists use to create their art?**
8. **What understandings must artists have to be successful?**

During Lesson Questions:

Video Artist Questions

1. **What methods did you observe the artist using in the video?**
2. **What observations did the artist make about his work? What information did the artist record?**
3. **What characteristics did the artist possess?**
4. **What tools did the artist use?**
5. **Did the artist's work remind you of anything familiar?**
6. **Where do you think the artist gets his inspiration, influences and ideas for his work?**
7. **How does the artist make use of pattern in his work?**
8. **Analyze how the artist gets from start to finish on a new piece of art?**
9. **What understandings did the artist have to have?**
10. **How did the artist use pattern to communicate in some way?**
11. **How did the artist make you feel?**

Evaluating the artists questions:

1. **What methods did you observe the artist using in the video?**
2. **What observations did the artist make about his work? What information did the artist record?**
3. **What characteristics did the artist possess?**
4. **What tools did the artist use?**
5. **Did the artist's work remind you of anything familiar?**
6. **Where do you think the artist gets his inspiration, influences and ideas for his work?**
7. **How does the artist make use of pattern in his work?**
8. **Analyze how the artist gets from start to finish on a new piece of art?**
9. **What understandings did the artist have to have?**
10. **How did the artist use pattern to communicate in some way?**
11. **How does the artist use pattern to convey feeling?**
12. **How does the artist use pattern to convey setting or location?**
13. **How does the artist use pattern to engage the audience?**
14. **How does the artist use pattern to win you over or get you to like his work?**
15. **How did the artist's work make you feel?**

Post Lesson Questions:

1. **What did you learn about being an artist?**
2. **What did you learn about being an artist mentor?**
3. **What strategies or methods did you use to choose a mentor?**
4. **How did you make inferences (conclusions reached using evidence or reasoning) for your mentor choices?**
5. **What rules did you impose on yourself and others working with you while choosing mentors?**
6. **How did you decide on or agree to these rules?**
7. **How important is pattern when working as an artist?**
8. **How do artists and artist mentors use pattern to inform prediction?**

DIFFERENTIATION *(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.*			
Content	**Process**	**Product**	**Learning Environment**
	Students work with an open-ended problem-solving experience. They use reasoning and evidence to create generalizations.	**Students have the opportunity to make observations and communicate at their individual level. (Pattern Topic)**	

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect - *This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.*

(20 min.) 12:30 – 12:50 As students enter the classroom, the teacher will be dressed as a stereotypical artist, i.e., hat and/or scarf, oversized white shirt with paint stains, artist easel & canvas, artist palette, and paint brush. Stereotypical French cafe' music will be playing in the background. https://www.youtube.com/watch?v=s6BuZOYboZM

Students will be instructed to create independent lists of what they know about artists using the pre-lesson questions. Teacher instructs students that they do not have to write the questions out, just number their responses. The list (posted on the board or overhead) should include:

1. **What do artists do?**
2. **What do artists wear?**
3. **What are some of the tools that an artist uses?**
4. **Where might artists work? Describe these spaces.**
5. **What characteristics should artists possess?**
6. **Are there rules artists must follow that impact their work?**
7. **What methods do artists use to create their art?**
8. **What understandings must artists have to be successful?**

After five minutes of listing, students are asked to share their thoughts with the class. The teacher will record responses so that the list is visible to the class under categories already written on the board. Students are asked to provide elaboration for their responses.

Explore - *In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.*

(90 min.) End 5 min break 12:50 – 2:15, break 2:15-2:20 The teacher introduces the video THE MAGIC MOMENT - Peter Dahmen The Amazing Artist! (6:55 min.)
http://thekidshouldseethis.com/post/the-magic-moment-the-work-of-paper-engineer-peter-dahmen

Students are instructed to watch the video by focusing on Peter Dahmen, the paper artist. They are to focus on what he does, what he advises, and what characteristics he possesses, as an observer.

After the video, students are referred back to their original list. Teacher asks: What other things would you add to this list? Teacher records student responses. The teacher then asks the during the lesson questions: (10 min.)

1. **What methods did you observe the artist using in the video?**
2. **What observations did the artist make about his work? What information did the artist record?**
3. **What characteristics did the artist possess?**
4. **What tools did the artist use?**
5. **Did the artist's work remind you of anything familiar?**
6. **Where do you think the artist gets his inspiration, influences and ideas for his work?**
   Looking for, but not limited to, from nature, the world around him, things that influence his senses and emotions, sight, sound, smell, taste & touch.
7. **How does the artist make use of pattern in his work?**
8. **Analyze how the artist gets from start to finish on a new piece of art?**
   Looking for, but not limited to, students to discuss that artists pose and answer questions as they create, build better understandings and draw conclusions in a variety of different ways; that artists make adjustments and self correct to meet the structure of the patterns they use which are the shapes and their special relationships.
9. **What understandings did the artist have to have?**
   Pattern, texture, structure, shape, spacial relationships, rotation, translation, reflection, combinations of the 3 (students may not use this terminology to describe these), tools
10. **Did the artist's work communicate in some way?**
11. **How did the artist's work make you feel?**

***At this point the teacher reminds the class the expectations for group work and class discussion.**

Students are then divided into groups of 2-3 students. The teacher tells the students that they are new artists seeking a mentor who (70 min) specializes in the use of pattern, and that as a committee, the class must pick the best choice, based on their knowledge of pattern, from their research and the videos they are about to watch. In addition, groups may research further and find their own artist to nominate to the class. The class discusses what a mentor does, and student groups are instructed that they must decide on their own criteria specific to what they are looking for, to help them pick that mentor and they must be prepared to defend their choice using their detailed observations, when we come back for class discussion. Students are allowed to make use of the questions we used while observing the paper artist and will accomplish their research online, each student may have a computer.

Street Foods Chinese Sugar Artist
https://www.youtube.com/watch?v=A4qvNu0vnvY **(4:14 min.)**

Pancake art - Lace Hearts + lace décor

https://www.youtube.com/watch?v=Dzeqn6_Ceos (1:42 min.)

Watermelon Art
https://www.youtube.com/watch?v=t0Nj8OywDMk (7:51 min.)

Pottery Master
https://www.youtube.com/watch?v=aib4HAXbuXc (6:00 min.)

Sand Wheel
https://www.youtube.com/watch?v=cxJIAA7prwk (5:20 min.)

Water Art
https://www.youtube.com/watch?v=X86uAWWZHi0 (14:03 min.) students will be advised to watch for 6-7 min.

Goldfish
https://www.youtube.com/watch?v=pyWe4rD_Rk0 (4:39 min.)

Explain - *Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.*

(20 min.) 2:20 – 2:40 After students have completed all research, the teacher provides time for groups to have one final meeting to draw conclusions based on all their observations. The teacher then asks groups to report their findings. Each group reports their conclusions and reasons for their conclusions. Students are encouraged to discuss conclusions of other groups and to contribute by questioning and adding their own conclusions. The class is instructed that they must come to a consensus and choose just one artist, and it is up to them how to make a final decision.

Elaborate —*Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways.*

(10 min.) 2:40 – 2:50 The teacher poses post-Lesson questions. Students respond to the questions orally. Answers are discussed and elaborated.

1. **What did you learn about being an artist?**
2. **What did you learn about being an artist mentor?**
3. **What strategies or methods did you use to choose a mentor?**
4. **How did you make inferences (conclusions reached using evidence or reasoning) for your mentor choices?**
5. **What rules did you impose on yourself and others working with you while choosing mentors?**
6. **How did you decide or agree on these rules?**
7. **How important is pattern when working as an artist?**
8. **How do artists and artist mentors use pattern to inform prediction?**

Evaluate: *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*

(50 min.) 2:50 – 3:40 STUDENTS CREATE THE 2ND PHASE OF THEIR TESSELLATION EVALUATION, REFLECTING A TESSELLATION (see attached) Student are also encouraged to add features and designs to these tessellations. (45 min.)
(5 min. clean up of materials) Students line up

If there are early finishers, they will have the opportunity to design tessellations online:
Tessellation Creator Online
http://illuminations.nctm.org/Activity.aspx?id=3533
Materials: clip boards, lined paper, writing utensils, expo markers, expo eraser, teacher computer, overhead, pattern blocks, Geometry Playground Exploring Tessellations packets, 3" x 5" index cards, rulers, scissors, blank paper or card stock, transparent tape, permanent colored markers/pens/pencils, tessellation examples

Name: _____

1. **What methods did you observe the artist using in the video?**

   _____
   _____
   _____
   _____

2. **What observations did the artist make about his work? What information did the artist record?**

   _____
   _____
   _____
   _____

3. **What characteristics did the artist possess?**

   _____
   _____
   _____
   _____

4. **What tools did the artist use?**

   _____
   _____
   _____
   _____

5. **Did the artist's work remind you of anything familiar?**

   _____
   _____
   _____
   _____

6. **Where do you think the artist gets his inspiration, influences and ideas for his work?**

   _____
   _____
   _____
   _____

7. **How does the artist make use of pattern in his work?**

   _____
   _____
   _____
   _____

8. **Analyze how the artist gets from start to finish on a new piece of art?**

   _____
   _____
   _____
   _____

9. **What understandings did the artist have to have?**

   _____
   _____
   _____
   _____

10. **Did the artist's work communicate in some way?**

_____
_____
_____
_____

11. **How did the artist's work make you feel?**

_____
_____
_____
_____

Links:
Street Foods Chinese Sugar Artist
https://www.youtube.com/watch?v=A4qvNu0vnvY **(4:14 min.)**

Pancake art - Lace Hearts + lace décor
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Goldfish
https://www.youtube.com/watch?v=pyWe4rD_Rk0 (4:39 min.)

Name: _____

POST LESSON QUESTIONS

1. **What did you learn about being an artist?**

_____
_____
_____
_____

2. **What did you learn about being an artist mentor?**

_____
_____
_____
_____

3. **What strategies or methods did you use to choose a mentor?**

_____
_____
_____
_____

4. **How did you make inferences (conclusions reached using evidence or reasoning) for your mentor choices?**

_____
_____
_____
_____

5. **What rules did you impose on yourself and others working with you while choosing a mentor?**

_____
_____
_____
_____

6. **How did you decide or agree on these rules?**

_____
_____

7. **How important is pattern when working as an artist? Explain.**

_____
_____
_____

8. **How do artists and artist mentors use pattern to inform predictions?**

_____
_____
_____

TEACHER NAME	Lesson #
Tealecia Fletcher	4

MODEL	CONTENT AREA	GRADE LEVEL
Questioning	Mathematics	6

CONCEPTUAL LENS	LESSON TOPIC
Patterns	Patterns/Tessellations

LEARNING OBJECTIVES *(from State/Local Curriculum)*

CCSS.MATH.CONTENT.K.G.1
DESCRIBE OBJECTS IN THE ENVIRONMENT USING NAMES OF SHAPES, AND DESCRIBE THE RELATIVE POSITION OF THESE OBJECTS.

CCSS.MATH.CONTENT.2.G.A.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles,

quadrilaterals, pentagons, and hexagons.

CCSS.MATH.CONTENT.4.G.A.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

CCSS.MATH.CONTENT.4.G.A.3

Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into

matching parts. Identify line-symmetric figures and draw lines of symmetry

G-CO Experiment with transformations in the plane.

G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-GMD Visualize relationships between two-dimensional and three-dimensional objects.

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects.

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	THE ESSENTIAL QUESTION *(What question will be asked to lead students to "uncover" the Essential Understanding)*
PATTERNS INFORM PREDICTIONS	***How do Patterns Inform Predictions?***

CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	PROCESS SKILLS (What will students be able to do as a result of this lesson?)
Students will know… **1. The vocabulary for describing polygons in relationship to each other.** **2. Fibonacci Numbers are a type of pattern called a sequence.** **3. That patterns inform predictions of numbers.** **4. That patterns of number predictions have direct relationships to nature.** **5. What a variable is in relationship to sequences.** **6. That sequences can repeat or cycle.**	**Students will be able to…** **1. Describe polygons in relationship to each other.** **2. Predict the next number in the Fibonacci sequence.** **3. Apply the number prediction skill to other sequences or lists of numbers.** **4. Apply patterns of numbers to items in nature like flowers or seashells.** **5. Start with a number n, apply a rule to n to get n'.** **6. Apply a rule to form any sequence.** **7. Generate longer or shorter sequences.**

GUIDING QUESTIONS

What questions will be asked to support instruction?
Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	During Lesson Questions:	Post Lesson Questions:
1. What is a sequence? If students struggle, teacher asks, what does sequential order mean? **2. What are the elements or items that make a sequence?** **3. What does it mean for items to be "in sequence"?** **4. How are sequences used/useful?**	***1. Envelope #1:*** *a) What pattern do you see in the order of the numbers?* *b) Using your knowledge of pattern, predict what will the next number be?* *c) What use of pattern do you see in the puzzle picture?* *d) Describe how does this pattern relate to special patterns,*	***The envelopes introduced you to some new special details of pattern.*** ***1. How can these be applied to patterns in art?*** ***2. How do these special patterns apply to nature?*** ***3. Where and how do these new patterns have applications in other places besides nature or art?***

5. What are some examples of sequences? **6. What is the relationship between sequence and pattern?**	*tessellations?* *e) What strategies did you use?* ***2. Envelope #2:*** a) Starting from one of the small squares, can you put them together so that every time you add a square you can create a rectangle? b) Generalize what these squares have to do with the Fibonacci number pattern in envelope #1? c) Describe if there is a pattern to the squares. d) Included in your envelope is a spiral. How does pattern apply to spirals? Lay the spiral on top of the squares you put together. e) Hypothesize how the pattern of the squares would help you draw the spiral? f) Where do you see spirals in your world? g) Do Fibonacci squares have a direct or indirect relationship to you? Decide which and describe how. ***3. Envelope #3:*** a) A pair of baby rabbits is put in an enclosed garden. Each pair of rabbits produces a new pair of rabbits every month. From the second month on, these rabbits also become productive. Using your sense of pattern, predict how many pairs of rabbits there will be in the garden after one year. b) Interpret how can you make use of the Fibonacci number pattern in relationship to the rabbits? c) How would you use pattern to predict how many rabbits there would be in another year? d) What strategies did you use? ***4. Envelope #4:*** a) Can you find a Fibonacci sequence in the Pascal Triangle? b) Using the patterns you see, predict the numbers for the next 2 rows of Pascal's Triangle. c) Write down as many sequences/patterns as you can find in the triangle. d) Compare the Pascal Triangle numbers to the Fibonacci numbers. e) How would you relate Pascal's Triangle to patterns in art? f) How would you relate Pascal's Triangle to patterns in nature? ***5. Envelope #5:*** a) Using a calculator, try to find a link between the Fibonacci number sequence and Phi. b) List each thing you try and explain why you tried it. ***6. Envelope #6:*** a) Illustrate how you think hailstones relate to patterns. b) Will every value you start with generate a sequence that eventually settles to 4,2,1,4,2,1,…? c) Make a recommendation whether there could be a sequence that never settles down to a repeating cycle at all.	***4. How do these patterns inform predictions?***

DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	Learning Environment
	1. For some tasks, students will work in groups in support of each other. **2. The envelope activities allow grouped students to work at their own pace.** **3. Use of questions which force students to think critically about the concept of pattern.** **4. The envelope activities differentiate in the way they are scaffolded in graduating order and according to the required exploration. For instance envelope number one could have just had the numbers for the sequence for students to try to put in order to discover how a Fibonacci sequence works, but there is the added layer of the numbers being color puzzle pieces for students who would struggle with ordering.**	**1. Because there are different levels of difficulty to creating tesselleations according to the technique that is used, students differentiate according to whether they pick to use translation, reflection, rotation or a mixture of the three for their performance task.**	**1. For the engage activity, groups are provided with a tri-fold divider which besides being used to keep the crewmembers from peeking, it helps support the focus during the activity for students who are easily distracted.**

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect - *This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.*

(20 min.) 12:30 – 12:50 Mission Control Activity (to review what students learned/used about polygons throughout the week.)

When students enter the room, they will be put into groups of 3, and assigned the role of astronaut versus mission controller and possibly given a costume to match their role. (Teacher is trying to locate affordable disposable hazmat jumpers for astronauts and lab coats for mission control.) There will be applicable space scenes & low sound/music playing. Each grouping will be provided a large tri-fold divider and a set of pattern blocks.

The teacher explains that the person with the pattern is **Mission Control** and all others on the other side of the tri-fold are **Space Ship Crewmembers.** The space ship crewmembers are on a mission and have encountered problems – they have only one-way communication with Mission Control! Which means crewmembers may not ask questions of mission control, they may only listen. In order to find their way home they must follow Mission Control's orders exactly to rebuild their panel of controls.

Looking at the "panel" of shapes, Mission Control carefully describes the position of the shapes using as much vocabulary as possible to assist the Crew in constructing the panel, which will enable them to return safely to earth. As mission control speaks, the Crewmembers listen and construct the panel using Mission Control's description. When time is up, crewmembers compare their control panel to that of Mission Control and then switch places so Mission Control is now a crewmember and crewmembers are now Mission Control. New Mission Controllers create their own design "panel" to direct the new Crewmember with and repeat the process.

Teacher displays/writes short versions of these rules on the board for all to see.

1. Mission Control on one side with the pattern/Crewmembers on the other side with the pattern blocks
2. Mission Control describes and may talk to all/Crewmembers listen and do not talk
3. Crewmembers try to build the pattern Mission Control is describing
4. When time is up, crewmembers compare their control panel to that of Mission Control and then switch places so Mission Control is now a crewmember and crewmembers are now Mission Control and repeat, but new Mission Control creates a new design "panel".

Explore - *In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.*

(50 min.) 12:50 – 1:40 Fibonacci Envelopes Activity: see attached.

Teacher asks the Pre-Lesson questions.

1. **What is a sequence? If students struggle, teacher asks, what does sequential order mean?**
2. **What are the elements or items that make a sequence?**
3. **What does it mean for items to be "in sequence"?**
4. **How are sequences used/useful?**
5. **What are some examples of sequences?**
6. **What is the relationship between sequence and pattern?**

Most students will not be familiar with the Fibonacci sequence to start with. The teacher introduces the lesson as a challenge for them to work in groups through a set of mysterious puzzles in envelopes numbered 1-6. Each envelope has one puzzle and groups must do them in order. Each puzzle also has a clue, but the group must agree that they're stuck before they can peek at it. After each completed puzzle, students answer the matching set of questions. When they've completed a puzzle, groups check with the teacher before moving on to the next envelope.

The teacher builds up the challenge by explaining that, if any group completes all five puzzles, they'll get a special sixth envelope with a problem that's never been solved. It's a version of the 'Hailstone Problem', which is a deceptively simple premise. *Students may not get to all the envelopes during the allotted time period, which is acceptable.*

Envelopes (Questions in red)

1) Fibonacci numbers - this sheet has a series of Fibonacci numbers, which need to be cut up before putting them in the first envelope. There are no instructions, so pupils need to work out what's going on and recognize the pattern and explain it. The background picture might help some students construct the puzzle picture. Upon completion, I explain that these are called Fibonacci numbers and they are very special for the rule you use to find the next number, while not giving away anything in later puzzles.
   - *a) What pattern do you see in the order of the numbers?*
   - *b) Using your knowledge of pattern, predict what will the next number be?*
   - *c) What use of pattern do you see in the puzzle picture?*
   - *d) Describe how does this pattern relate to special patterns, tessellations?*
   - *e) What strategies did you use?*

2) Fibonacci squares - has a series of squares which will be cut up before putting them in the envelope. Students should recognize the pattern, build up the shape and explain what's going on. There's also a spiral to help explore how this relates to the sequence of squares.
   - a) Starting from one of the small squares, can you put them together so that every time you add a square you can create a rectangle?
   - b) Generalize what these squares have to do with the Fibonacci number pattern in envelope #1?

c) Describe if there is a pattern to the squares.
d) Included in your envelope is a spiral. How does pattern apply to spirals? Lay the spiral on top of the squares you put together.
e) Hypothesize how the pattern of the squares would help you draw the spiral?
f) Where do you see spirals in your world?
g) Do Fibonacci squares have a direct or indirect relationship to you? Decide which and describe how.

3) Rabbits

a) A pair of baby rabbits is put in an enclosed garden. Each pair of rabbits produces a new pair of rabbits every month. From the second month on, these rabbits also become productive. Using your sense of pattern, predict how many pairs of rabbits there will be in the garden after one year.
b) Interpret how can you make use of the Fibonacci number pattern in relationship to the rabbits?
c) How would you use pattern to predict how many rabbits there would be in another year?
d) What strategies did you use?

4) Pascal's triangle - for students to recognize various number sequences.

a) Can you find a Fibonacci sequence in the Pascal Triangle?
b) Using the patterns you see, predict the numbers for the next 2 rows of Pascal's Triangle.
c) Write down as many sequences/patterns as you can find in the triangle.
d) Compare the Pascal Triangle numbers to the Fibonacci numbers.
e) How would you relate Pascal's Triangle to patterns in art?
f) How would you relate Pascal's Triangle to patterns in nature?

5) The Golden ratio and phi - for more advanced students to work out the connection between phi and Fibonacci numbers.

a) Using a calculator, try to find a link between the Fibonacci number sequence and Phi.
b) List each thing you try and explain why you tried it.

6) The Hailstone Problem - for those who make it that far!

a) Illustrate how you think hailstones relate to patterns.
b) Will every value you start with generate a sequence that eventually settles to 4,2,1,4,2,1,…?
c) Make a recommendation whether there could be a sequence that never settles down to a repeating cycle at all.

There's a website about every aspect of Fibonacci numbers at: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/
- And more about the 'Hailstone Problem' at: http://blog.functionalfun.net/2008/07/project-euler-problem-14-hailstone.html . I will decide if students need to access these sites for support or if we explore the topics further as a class.

Explain - *Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.*

Class comes together to discuss the envelope activities.
Teacher asks groups to share their during the lesson question conclusions for each envelope. At this point the teacher will elaborate.

Elaborate —*Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways*

The envelopes introduced you to some new special details of pattern.

1. *How can these be applied to patterns in art?*
2. *How do these special patterns apply to nature?*
3. *Where and how do these new patterns have applications in other places besides nature or art?*
4. ***How do these patterns inform predictions?***

(95 min.) 1:40 – 1:50 (10 min. break) 1:50 – 3:15 Performance Task: Commissioned Artist Activity explained:

A very prestigious art museum is having a contest for the cover art for their next advertising campaign that includes a brochure, posters and newspapers. This is a great opportunity for new artists like you to get exposure to increase your sales and chances for success. You are challenged to meet the requirements. The winning piece will be the one that is most representative of the theme and meets all requirements. The theme is pattern and the museum committee will be judging all entrants on the following criteria:

1. *Artist must include the written plan they followed to create their work of art.*
2. *Artist must make use of several types of patterns, including the three types of tessellations and at least one special pattern like Fibonacci or the Golden Ratio.*
3. *Artist must include a description of the inspiration for their art.*
4. *Artist must include a connection to nature and/or the real world.*
5. *Artist is challenged to be creative with use of pattern.*
6. *Art must be original.*

Honing skill activity. Teacher led, STUDENTS CREATE THE 3rd. PHASE OF THEIR TESSELLATION, MAKING A ROTATION TESSELLATION and helps students review their tessellation skills from the week. (15 min.) (see attached) (1:50 – 2:05)

(30 min.) 3:15 – 3:45 Evaluate: *This phase assesses both learning and teaching and can use a wide variety of informal and formal*

assessment strategies.

Teacher arranges student artwork around the room. Teacher tells students they are tasked to act as the committee judging the artwork and go around the room making detailed observations. After observations of each piece of work, the teacher makes student groups of no more than 3 students per group who then meet to record their findings and make their generalizations. The teacher then provides time for groups to have one final whole class meeting to draw conclusions based on all the observations. The teacher asks students to report their findings. Each group reports their conclusions and reasons for their conclusions. Students are encouraged to discuss conclusions of other groups and to contribute by questioning and adding their own conclusions. Finally, the class is tasked to award each piece of art a category, no duplicates – each art piece may be awarded one catergory.

Committees meet and decide the following awards:

15. *Best use of pattern.*
16. *Best use of color.*
17. *Best-written submission.*
18. *Best connection to nature/ the real world.*
19. *Best use of tessellations.*
20. *Most symmetrical.*
21. *Most Entertaining.*
22. *Most Interesting. (Makes you think)*
23. *Best use of a variety of media.*
24. *Most original.*
25. *Most creative.*
26. *Best interpretation of the pattern theme.*
27. *Quality of composition/work.*
28. *Best overall clear emphasis of the theme pattern.*

Lesson 4 Envelope Questions

1. ***Envelope #1:***
   a. What pattern do you see in the order of the numbers?
   _____
   _____
   b. Using your knowledge of pattern, predict what will the next number be?
   _____
   c. What use of pattern do you see in the puzzle picture?
   _____
   _____
   d. Describe how does this pattern relate to special patterns, tessellations?
   _____
   _____
   e. What strategies did you use?
   _____
   _____
   _____

2. ***Envelope #2:***
   a. Starting from one of the small squares, can you put them together so that every time you add a square you can create a rectangle?
   _____
   _____
   b. Generalize what these squares have to do with the Fibonacci number pattern in envelope #1?
   _____
   _____
   _____
   c. Describe if there is a pattern to the squares.
   _____
   _____
   _____
   d. Included in your envelope is a spiral. How does pattern apply to spirals? Lay the spiral on top of the squares you put together.
   _____
   _____
   _____
   e. Hypothesize how the pattern of the squares would help you draw the spiral?
   _____
   _____
   _____
   f. Where do you see spirals in your world?
   _____
   _____
   _____
   g. Do Fibonacci squares have a direct or indirect relationship to you? Decide which and describe how.
   _____
   _____
   _____

3. ***Envelope #3:***
   A pair of baby rabbits is put in an enclosed garden. Each pair of rabbits produces a new pair of rabbits every month. From the second month on, these rabbits also become productive. Using your sense of pattern, predict how many pairs of rabbits there will be in the garden after one year.
   _____
   a. Interpret how can you make use of the Fibonacci number pattern in relationship to the rabbits?

_____
_____
_____

b. How would you use pattern to predict how many rabbits there would be in another year?

_____
_____
_____

c. What strategies did you use?

_____
_____
_____

4. ***Envelope #4:***

a. Can you find a Fibonacci sequence in the Pascal Triangle?

_____
_____
_____

b. Using the patterns you see, predict the numbers for the next 2 rows of Pascal's Triangle.

_____
_____

c. Write down as many sequences/patterns as you can find in the triangle.

_____
_____
_____

d. Compare the Pascal Triangle numbers to the Fibonacci numbers.

_____
_____
_____

e. How would you relate Pascal's Triangle to patterns in art?

_____
_____
_____

f. How would you relate Pascal's Triangle to patterns in nature?

_____
_____
_____

5. ***Envelope #5:***

a. Using a calculator, try to find a link between the Fibonacci number sequence and Phi.

_____
_____
_____

b. List each thing you try and explain why you tried it.

_____
_____
_____

6. ***Envelope #6:***

a. Illustrate how you think hailstones relate to patterns.

_____
_____
_____

b. Will every value you start with generate a sequence that eventually settles to 4,2,1,4,2,1,…?

_____
_____
_____

c. Make a recommendation whether there could be a sequence that never settles down to a repeating cycle at all.

_____
_____

You will have:

6 mathematical puzzles - one in each envelope
The envelopes are numbered.
You have 5 envelopes are on your desk - one is missing!

In each envelope you will find:

1 x master colour copy of the question
4 x black and white copies to write on if you need to.

What to do:

Work through each envelope in order and solve each puzzle.
When you've finished one puzzle, check with a teacher before going on to the next.

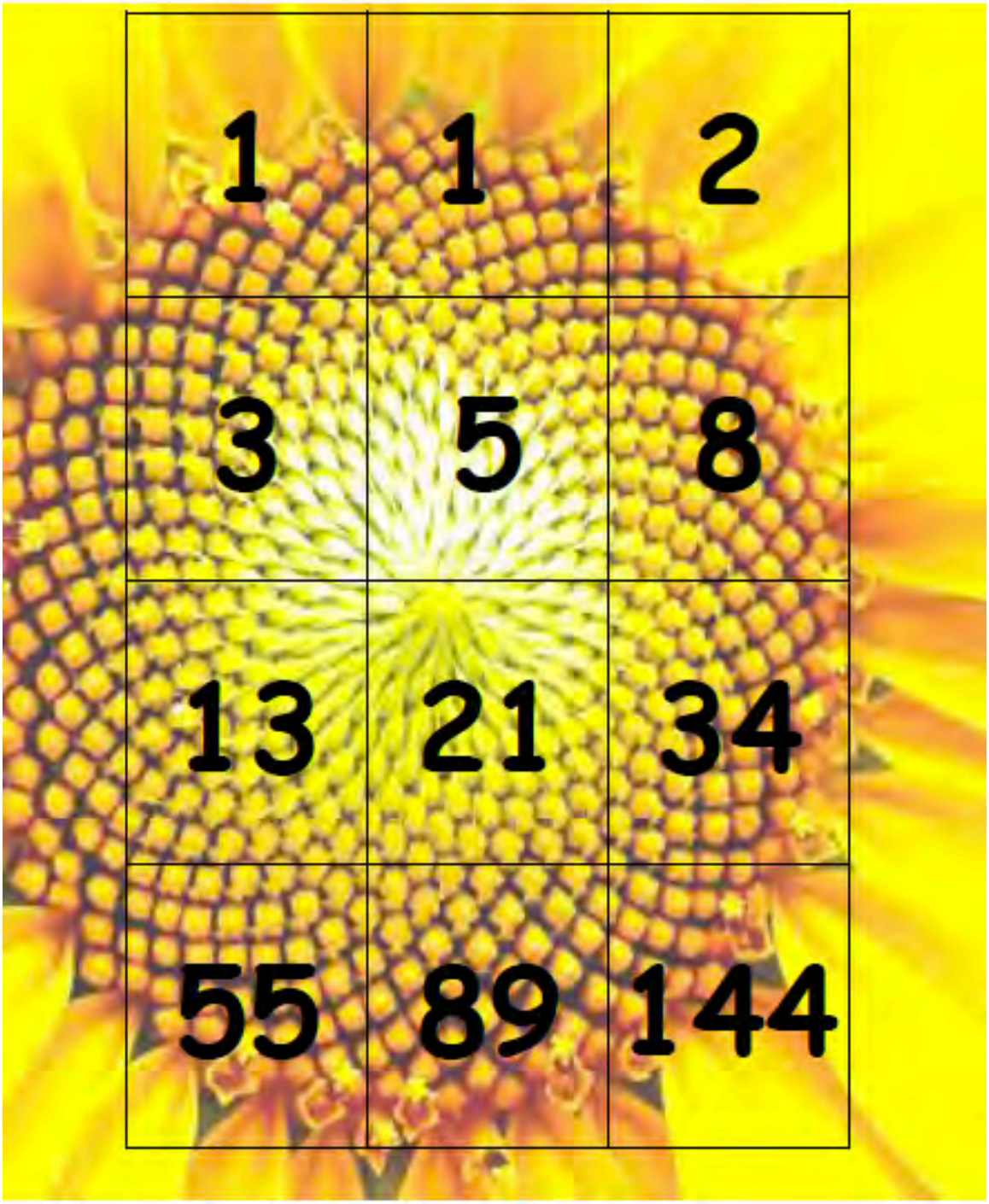
You also have:

Clues for each puzzle.
Only open a clue when the whole group has run out of ideas!

Pack away each puzzle when you have finished with it.

Remember - talk amongst yourselves
BUT ONLY ABOUT THE MATHS!

Puzzle 1

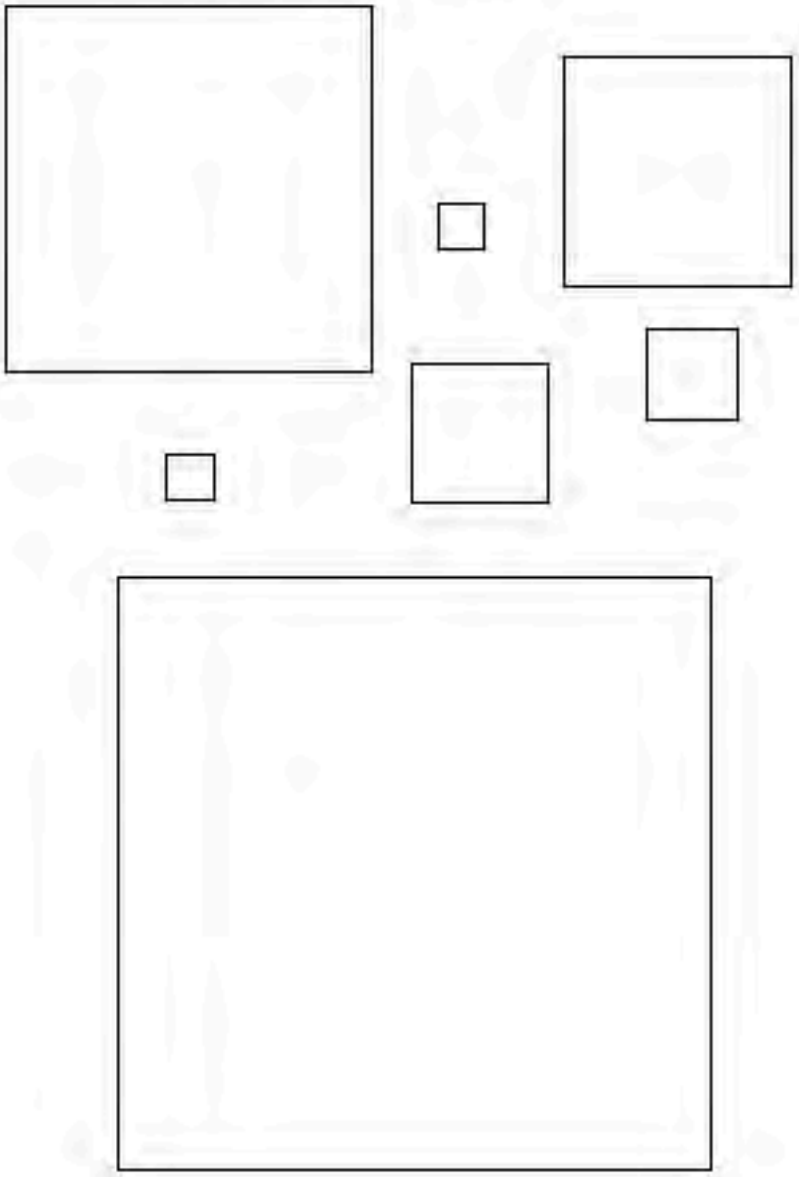


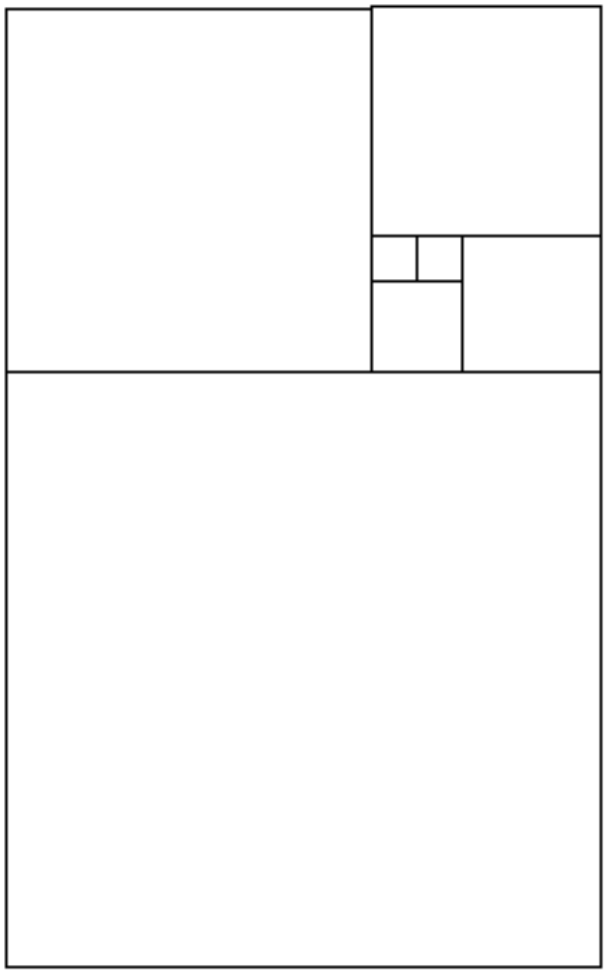
Puzzle 2:

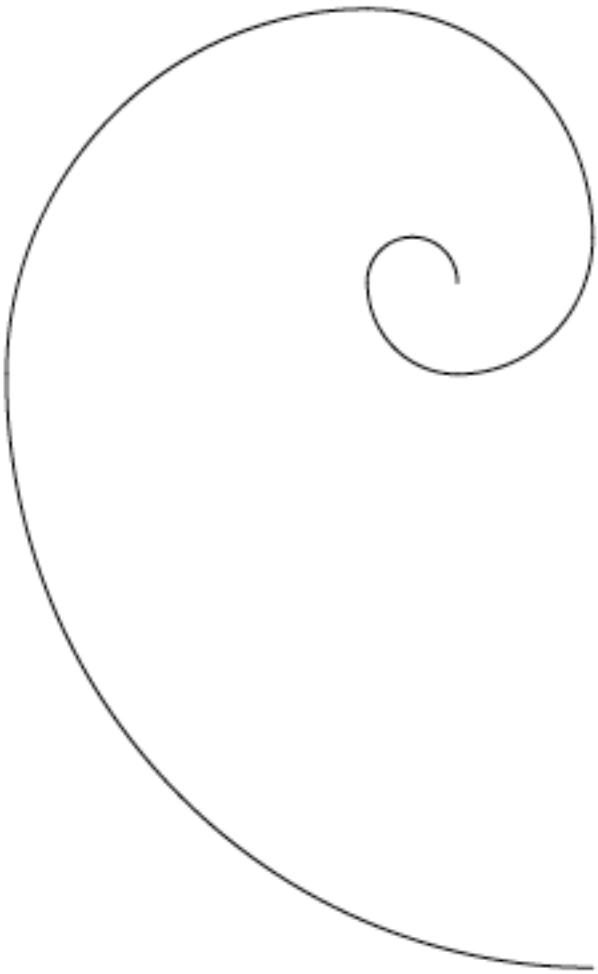
1. What do these squares have to do with Fibonacci numbers?

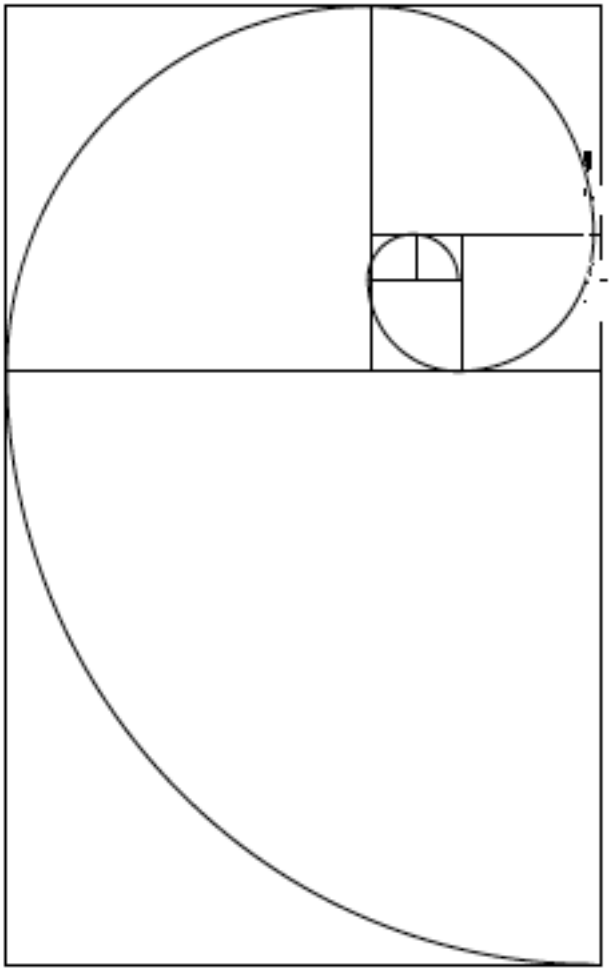
2. Starting from one of the small squares, can you put them together so that every time you add a square you can create a rectangle?

3. How do these squares help you draw a spiral?









Puzzle 3

Fibonacci rabbits

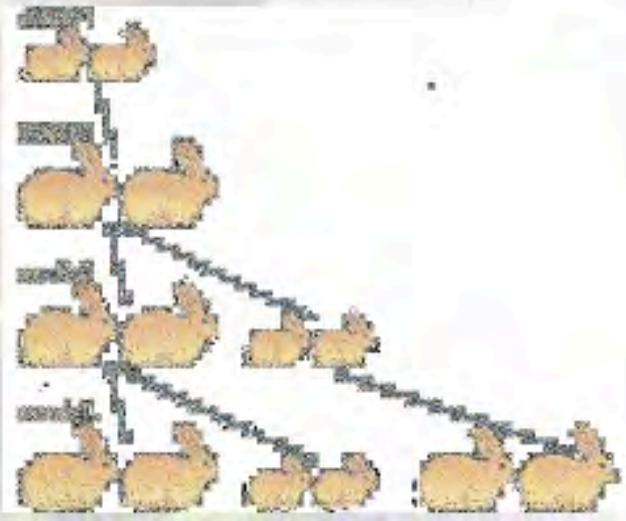
A pair of baby rabbits are put in an enclosed garden.

Each pair of rabbits produces a new pair of rabbits every month.

From the second month on, these rabbits also become productive.

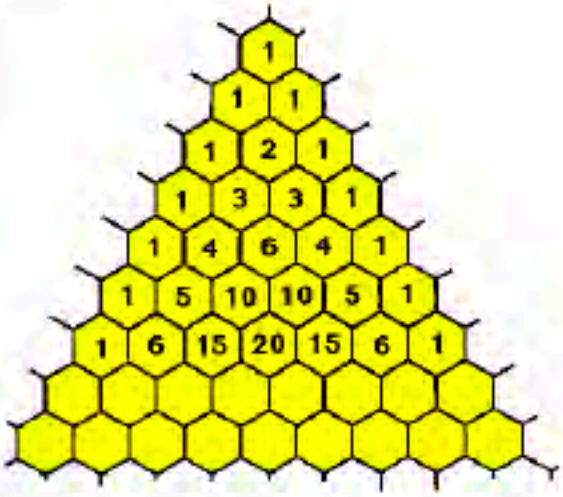
How many pairs of rabbits will there be in the garden after one year?

Continue the diagram and find out the number of pairs of rabbits in the Fibonacci garden after one year.



Puzzle 4

Pascal Triangle



1. Complete the next 2 rows of this Pascal triangle.

2. Write down as many sequences as you can find in the triangle.

3. Can you find a Fibonacci sequence in the Pascal triangle?

Puzzle 5

Φ = 1.6180339887498948482.....

The Golden Ratio is a special number that goes on forever, without repeating, like Pi (π).

It is often written as the Greek letter Phi (Φ)

Use a calculator to find a link between the Fibonacci sequence and Phi.

Explain what you found

Clue to puzzle 1:

Put the numbered cards in order - the picture might help.
Look at the sequence you get.
Can you work out the rule that links the numbers?

Clue to puzzle 2:

Look at the side length of each square.
Can you work out the rule that links them?
How does it compare with the number sequences in puzzle 1?

Clue to puzzle 3:

Consider the number of pairs of rabbits each month.
How many are there at the beginning?
How many after a month?
After 2 months?
After 3 months?
How do the pairs link to Fibonacci?

Clue to puzzle 4:

Consider drawing lines across the hive.

Is there a link between each number and the ones above it?

Clue to puzzle 5:

Use your calculator to divide each Fibonacci term by the previous one:
$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$, and so on.... How does this relate to the value of phi?

Puzzle 6

Mathematical mysteries: Hailstone sequences

This problem is easy to describe but it is one of mathematics' unsolved problems.

Starting with any positive integer n, form a sequence in the following way:

If n is even, divide it by 2 to give n' = n/2.

If n is odd, multiply it by 3 and add 1 to give n' = 3n + 1.

Then take n' as the new starting number and repeat the process. For example:

n = 5 gives the sequence 5, 16, 8, 4, 2, 1, 4, 2, 1,...

n = 11 gives the sequence

11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1,...

These are sometimes called "Hailstone sequences" because they go up and down just like a hailstone in a cloud before crashing to Earth – the endless cycle 4, 2, 1, 4, 2, 1.

It seems from experiment that such a sequence will always eventually end in this repeating cycle 4, 2, 1, 4, 2, 1,... and so on.

But some values for N generate many values before the repeating cycle begins. For example, try starting with n = 27. See if you can find starting values that generate even longer sequences.

The unsolved problem is to prove that every starting value will generate a sequence that eventually settles to 4, 2, 1, 4, 2, 1,...?

Could there be a sequence that never settles down to a repeating cycle at all?

Good luck!

Resources for Teachers and Students

1. Artful Mathematics: The Heritage of M. C. Escher, Celebrating ...
   www.ams.org/notices/200304/fea-**escher**.pdf
   American Mathematical Society
   Apr 2, 2003
   Very informative for the geometry in his art and background behind specific pieces.
2. "M.C. Escher." - *The Official Website*. N.p., n.d. Web. 04 Aug. 2016.
   This is a one stop shop for everything about the artist; his life, including videos, and all of his art from sculptures to paintings.
3. Common Core in Action: 10 Visual Literacy Strategies | Edutopiawww.edutopia.org/blog/ccia-10-visual-literacy-strategies-**todd-finley**
   Edutopia Feb 19, 2014
   Shows the purpose of visual literacy (VL), to explicitly teach a collection of competencies that will help students think through, think about and think with pictures.
4. **America's Favorite Card Games®**
   http://www.setgame.com/
   This is the website for purchasing the game, rules for playing the game and receiving support for questions.
5. **Tessellation: The Geometry of Tiles, Honeycombs and M.C. Escher**
   www.livescience.com › Strange News
   Mar 3, 2015
   This website puts tessellations in real world context with lots of visuals and explanations.
6. **Cool math Lessons**
   http://www.coolmath.com/lesson-tessellations-1
   This website has an awesome step by step lesson on what tessellations are, the vocabulary terms that are clearly explained, color examples of each type, what shapes will and will not tessellate and how to properly describe combination tessellations.
7. Fathauer, Robert. (2014). ***Tessellations Around the World.*** Phoenix, AZ: Math + Art = Learning^2 Publishing.
   Great resource for tessellation patterns in relationship to parts of the world. There are questions and explorations for each tessellation.
8. ***Geometry Playground Activities Grades 6-8***
   Retrieved from
   https://www.exploratorium.edu/geometryplayground/Activities/GP_Activities_6-8/ExploringTessellations_%206-8_v4.pdf
   Shows how to create tessellations using cut tiles with rotation, translation and reflection.
9. ***Math Jingle***
   Retrieved from *http://www.hbschool.com/jingles/jingles_all/35i_repeat.html*
   Song with pattern/repetition, about patterns.
10. ***Bond of Union***
    Retrieved from *http://www.mcescher.com/gallery/recognition-success/bond-of-union/*
    This is Escher's art for the Visual Thinking Strategy.

11. ***Other Tessellation Artists***

    Andrew Crompton Tessellation Sculptures & paintings
    http://www.cromp.com/pages/tess1.html

    Wolter Schraa fractal art
    https://wolter.home.xs4all.nl/index.html

    David E. Joyce Hyperbolic Tessellations
    http://aleph0.clarku.edu/~djoyce/poincare/poincare.html

    Bruce Bilney Animal Tessellations
    http://www.ozzigami.com.au/tessellations.html

    Roger Penrose (Penrose triangle and stairs)
    http://www.break.com/article/how-the-rit-stairwell-illusion-works-2440836

12. ***The Kid Should See This: The Magic Moment: Work by paper engineer Peter Dahmen***
    http://thekidshouldseethis.com/post/the-magic-moment-the-work-of-paper-engineer-peter-dahmen

12. Websites for Videos of Artists Who Use Unusual Materials:

Street Foods Chinese Sugar Artist
https://www.youtube.com/watch?v=A4qvNu0vnvY

Pancake art - Lace Hearts + lace décor
https://www.youtube.com/watch?v=Dzeqn6_Ceos

Watermelon Art
https://www.youtube.com/watch?v=t0Nj8OywDMk

Pottery Master
https://www.youtube.com/watch?v=aib4HAXbuXc

Sand Wheel
https://www.youtube.com/watch?v=cxJIAA7prwk

Water Art
https://www.youtube.com/watch?v=X86uAWWZHi0.

Goldfish
https://www.youtube.com/watch?v=pyWe4rD_Rk0

13. **Tessellation Creator Online**
    http://illuminations.nctm.org/Activity.aspx?id=3533
    This is an interactive website for creating digital art.

14. Fibonacci Numbers and the Golden Section
    http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/

http://blog.functionalfun.net/2008/07/project-euler-problem-14-hailstone.html.

These are everything about these numbers and the hail stone problem, including fun facts, podcasts and links to a ton of fun math topics.