TRIGONOMETRY TESTED

By Brooke Parise

Grade 8

August 2, 2015

INTRODUCTION

Trigonometry is a large part of Algebra and Geometry. The study of right triangles can be applied in architecture, biology, engineering, computer design, and numerous other careers. Pythagoras found the study of right triangles to be useful in the 500s B.C, and this topic's importance has lasted through the ages. Because of Trigonometry's importance in the world, most middle and high school curriculums have a large focus on the topic. In Winston Salem/Forsyth County Schools, there are many units dedicated to Trigonometry.

* Learning Objective 8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

* Learning Objective 8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real world and mathematical problems.

* Learning Objective G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
* Learning Objective G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
* Learning Objective G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

In order for gifted learners to understand Trigonometry, differentiation is necessary. The **content** for gifted learners can be more in depth than other learners. For example, regular students are taught Trigonometry at a slower pace. They are taught the Pythagorean one year, then the Sine, Cosine, and Tangent ratios another year. Using the inverses of Sine, Cosine, and Tangent are techniques that few regular learners are ready for, and when taught the third year of high school math, they need much remediation on the topic. Gifted learners may be taught the Pythagorean Theorem in a short period of time, and then they are ready to move onto the Sine, Cosine, and Tangent ratios. The inverses of Sine, Cosine, and Tangent are still tricky for them, but they learn it and understand the theory behind it in most cases.

The **process** for teaching gifted students Trigonometry is much different than teaching regular learners. Gifted students can be taught the rules very quickly, and then they are ready to apply them to real world situations. Instead of the teacher doing many example problems with the students, and then the students practicing with many similar problems on their own, gifted children can usually view one example and then apply it to the world. The students who do not understand the concepts as quickly benefit from a few more teacher-led examples, then some practice problems on their own. After they have done this, then they are ready to apply Trigonometric concepts to the real world.

The **product** for this unit is the Moravian Star and a letter to a company giving the star's surface area and volume (volume of the points only). This star is not easy to make. The directions use technical language. The volume and surface area are very difficult to calculate, as the students have to use Trigonometry to find the height of each pyramid and the height of each pyramid's lateral face. Simply knowing HOW to draw a two-dimensional triangle within each pyramid to find each of these heights requires a three-dimensional thinker. This product can be differentiated in that the teacher can simply give the directions to some students, and those students can find conclusions for each of their questions on their own. Other students may need guidance through the math, setting up the two-dimensional triangles, or reading the technical directions. The highly gifted learners may get creative with their math and even invent ways to solve the problem that the teacher did not think of.

The **learning environment** for gifted learners may vary more than for non-gifted learners. When challenged, gifted students tend to focus intently, thus giving them the freedom to not need the monitoring of a teacher. Some students may work in the hallway or in the cafeteria. Other students who do not understand the directions or the concepts as quickly may get frustrated and give up easily. These students may need more monitoring, instruction, and encouragement from the teacher, so they may stay in the classroom with the teacher.

Gifted learners come in a variety of packages; you really cannot judge a book by its cover. Though gifted students may come from any socioeconomic background, I have found that

the more educated parent advocates for their child to be tested for giftedness more often than the parents who have not attended college or work in blue collar careers. Because of this, the actual gifted classroom tends to be predominantly children of middle class or upper class parents. This is good, in that the parents are involved in motivating their children to study and participate in extracurricular activities, as they see the vital importance in both. But there is a great need for children whose parents do not know the benefits their children could be receiving through gifted education simply because they have not experienced it themselves. There is much unlocked potential in many students who are not in the gifted classroom.

In the gifted classroom, you may find an extraordinary number of students who are involved in orchestra, band, more than one sport, and a variety of academic clubs, such as Lego Robotics, National Academic League, or Model U.N, to name a few. These students vary in their interests, but they seem to have one thing in common: they take their interests very seriously and put much time and effort into them. Competition and succeeding are very important to gifted children, and their parents seem to be very committed to helping their children succeed at their interests. The parents realize that the student accelerates in the classroom, but they also have passions outside of the classroom that help them to be whole people.

GOALS AND OUTCOMES

The goals of this unit include a content, process, and concept goal. The **content goals** are established by the Winston Salem/Forsyth County Administrators. For this unit, the goals are: applying the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions, knowing the formulas for the volumes of cones, cylinders, and spheres and use them to solve real world and mathematical problems understanding the side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

The **process goal** for this unit is different for each student. Each student needs to improve upon the process that they are currently employing. For example, if a student needs instruction, examples, and then many more teacher-led examples in order to understand a new concept, then the goal for that student is to try to practice the new concept without so much teacher intervention. If a student need much help through technical reading, then the goal for that student is to try to figure out the technical reading on his own, and then ask the teacher if he is on the right track. The goal for everyone is the same, in that; they each are learning to be more and more independent with their learning, processing, and applying of the math.

The **concept** for this unit is how investigation informs design. Whether the students are investigating how to find the volume and surface area of the Moravian Star while not given all of the necessary measurements or they are actually doing technical reading in order to create their own star, they are having to investigate. Again, with the goal being to foster independent learning, the more independent they become, the more investigating they will pursue, as opposed to always being directed by a teacher.

ASSESSMENT

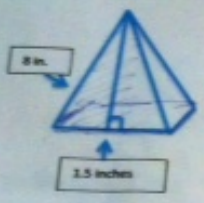
The student assessment for this unit consists of ****parts. First, the students will complete the Moravian Star Surface area and Volume worksheet. This worksheet is set up to guide students through their findings based on the fact that they already know how to use the Pythagorean Theorem, find the volume and surface area of pyramids, and visualize three dimensional shapes and the segments found within those shapes. For highly gifted students, this worksheet would be unnecessary. The teacher would simply have the students find the components of the project that "the company" wanted them to find (the students would deduce that these were the volume and surface area), and they would find the solutions without a worksheet to lead them. The third category of students would be those who may not be familiar with the formulas more three dimensional visualization. In this case, the teacher can guide them through the worksheet using questioning. Whether the group needs assistance or not, all students must turn in their report for "the company" with the accurate measurement.

Miss Parise

Moravian Star

Surface Area: 518.8 in^2
Volume: (6 pt. only) 190.08 in^3

Large points: 18 of them (4-sided base)



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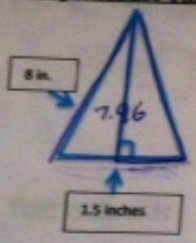
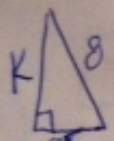
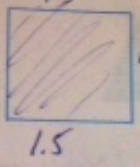




$K = 7.96$

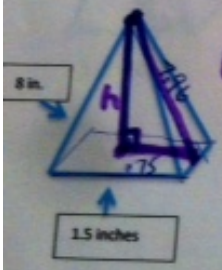
$a^2+b^2=c^2$
$K^2+.75^2=8^2$
$K^2+.56=64$
$-.56 \quad -.56$
$K^2 = 63.44$

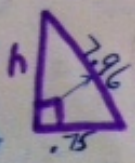
Base



Area = $l \cdot w$
$= (1.5)(1.5)$
$A = 2.25 in^2$

Height of Pyramid





$a^2+b^2=c^2$
$h^2+.75^2=7.96^2$
$-.75^2 \quad -.75^2$
$h^2 = 63.36 - .56$
$h^2 = 62.8$
$h = 7.92$

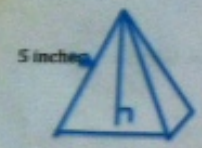
Volume = $\frac{1}{3}$ (Base's Area)(height) *one lg. pt.

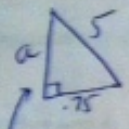
$\frac{1}{3}(2.25)(7.92) = 5.94 in^3$

Surface Area = $\left(\frac{b \cdot h}{2}\right)(4) = \left(\frac{1.5 * 7.96}{2}\right)4$

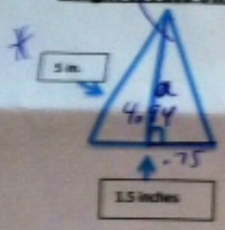
$SA = 23.88 in^2$
*of 1 lg. pt.

Small points: 8 of them (3-sided base)





Height of Face Only



$a^2+b^2=c^2$

$a^2+.75^2=5^2$

$-.75^2 \quad -.75^2$

$5^2-.75^2=a^2$

$25-.56=a^2$

$\sqrt{24.44}=\sqrt{a^2}$

$4.94=a$

Base

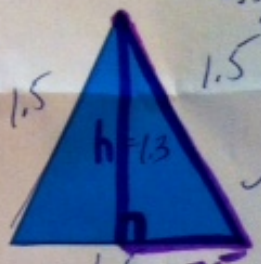
$h^2+.75^2=1.5^2$

$h^2+.5625=2.25$

$-.5625 \quad -.5625$

$\sqrt{h^2}=\sqrt{1.69}$

$h=1.3$

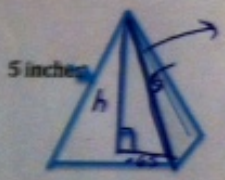


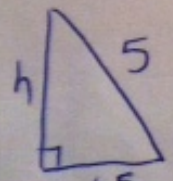
$\text{Area}=\frac{b\cdot h}{2}$

$=\frac{(1.5)(1.3)}{2}$

$A=.98$

Height of Pyramid





$h^2+.65^2=5^2$

$h^2+.42=25$

$-.42 \quad -.42$

$\sqrt{h^2}=\sqrt{24.58}$

$h=4.96$

Volume $\frac{1}{3}(B)(h)$

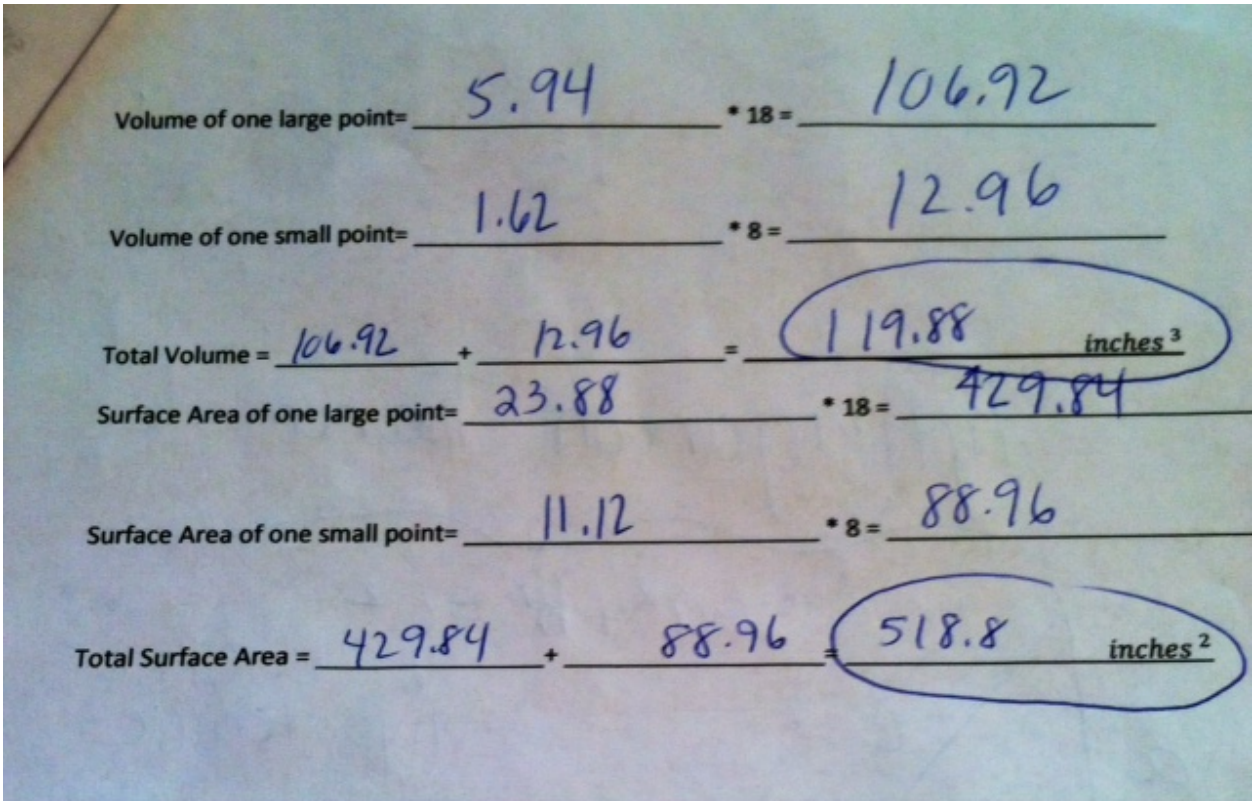
$\frac{1}{3}(.98)(4.96)=1.62$

Surface Area

$=(\text{area of face})(3)$

$=\left(\frac{(1.5)(4.94)}{2}\right)(3)$

$SA=11.12$ for 1 small pt.



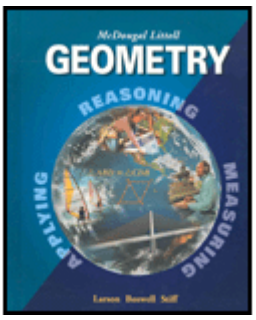
The second assessment tool is the Moravian Star. Grading is based on having 18 large points measuring 8 inches on each edge (worth 45 points), 8 small points measuring 5 inches on each edge (worth 45 points), and neatness (worth 10 points).

After the unit, the students will be tested on finding the volume and surface areas of cones. The component that makes this gifted in nature is the fact that the heights of the pyramids will not be given. The students will have to use their three dimensional thinking to find the height of the pyramids, as well as the lateral faces' heights. The Pythagorean

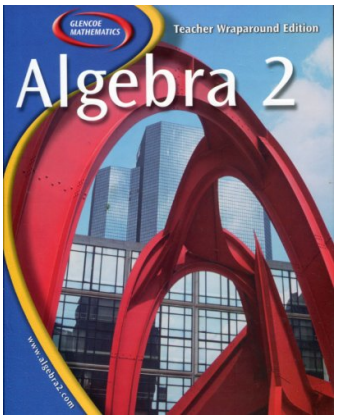
Theorem will also be tested on this assessment, as its use is necessary for finding the heights.

UNIT RESOURCES

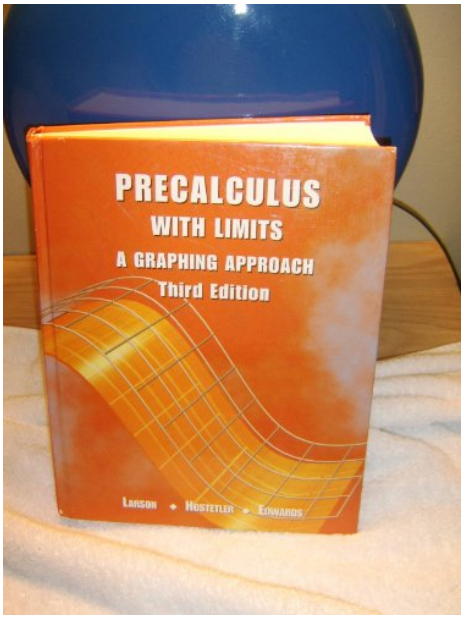
The main resource for this unit is McDougal Littell's "Geometry" Pages 535-541 explain in detail the Pythagorean Theorem and give many example and review problems. Resources are given on the volume of pyramids, the surface area of pyramids, and the height, lateral faces' heights and edge lengths, as well (pages 735-775).



After students have mastered solving for the lengths of sides of a right triangle using the Pythagorean Theorem, they can move on to using the Trigonometric functions Sine, Cosine, and Tangent to solve for the angles, too. Glencoe Mathematics' "Algebra 2" is a superb resource to use in teaching this material, and it even gives challenge application problems. (pages 701-771)



Once the students have mastered using Trigonometry to solve for right trinagles, they may use the Law of Sines and Law of Cosines to solve for non-right triangles. "Precalculus with Limits" by Larson, Hostetler, and Edwards is a great resource for these lessons. (Pages 424-437)



Surely, gifted students are not simply satisfied learning formulas and applying them to simple two-dimensional shapes, in which all of their needed measurements are given. These students require the challenge of having to find needed measurements using complicated formulas, and having to draw two-dimensional pictures within three-dimensional shapes in order to find those measurements. Investigation informs design with every aspect of life, and these students will have the visual reminder of the Moravian Star with them to always remind them of this realization.

TEACHER NAME	Lesson #
Brooke Parise	1 (This is the 1st lesson for STEP summer school.)

MODEL	CONTENT AREA	GRADE LEVEL
Questioning	8th Grade Math	8

CONCEPTUAL LENS	LESSON TOPIC
Investigations	Geometric Measurements

LEARNING OBJECTIVES *(from State/Local Curriculum)*

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real world and mathematical problems.

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	THE ESSENTIAL QUESTION *(What question will be asked to lead students to "uncover" the Essential Understanding)*
Investigation Informs Design	How does investigation inform design?

CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	PROCESS SKILLS (What will students be able to do as a result of this lesson?)
The students will know: - Surface Area of A Pyramid= (sum of lateral faces' areas)+ (base area) - Pythagorean Theorem : $a^2 + b^2 = c^2$ - Area of a Triangle =$\frac{1}{2}(base)(heigt)$ - Volume of Pyramid = =$\frac{1}{3}(base\ area)(height)$ - Area of a square = (length)(width) - 1 fluid ounce = 1.8 cubic inches	The students will analyze the Moravian star and break it down into its individual parts in order to find the surface area and volume of the entire structure. They will use their judgment to conclude that the base area of each pyramid is unnecessary in finding the surface area of the star. The students will conclude that they are not given enough information in order to find the volume and surface area, and they will decide when to ask for the teacher's help. Having been taught the Pythagorean Theorem, the students will decide when its use is necessary. (They will conclude that the Pythagorean Theorem is necessary to find the height of each pyramid, as well as the lateral faces' heights.) The students will calculate the volume and surface area of pyramids. In order to make the final presentation to the company, the students will conclude that they must convert their volume units, which are$inches^3$, to ounces. They will decide to use the internet for the conversion ratio, as this conversion is not common knowledge.

GUIDING QUESTIONS

What questions will be asked to support instruction?

Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	During Lesson Questions:	Post Lesson Questions:

The teacher will say: I have a design problem that I'd like your help investigating. I am working with a company to make a prototype of a Moravian Star to eventually make and sell, but they need my exact dimensions by next week. I know some of the dimensions, but in order to make the prototypes, they need to know how much material each star will use. Another issue is that after they make the stars, they are thinking of filling them with a glowing fluid. So I need to tell them how many fluid ounces each star could hold. (The teacher will put the students into groups or three to discuss and draw a plan. The groups will be random, as this is the first day of class, and the ability level and work ethic of each student is unknown. The students will use the internet to find any information or formulas that they may need.) The teacher will say: Let me show you the parts of the star that I know already, and I'd like us to come up with what other measurements I need in order to give the company what they want. (The teacher will question only when necessary to guide them to their previous knowledge.)	-The teacher will ask each group what they came up with in order to satisfy the prototype company's order. The teacher will only insert questions to guide them when they are stumped.	- After every group has presented, the teacher will ask the students what they think the best answers from each group. She will then write them on a final, large paper at the front of the room. This will be the final presentation that she will give to the prototype company. -The teacher will fill a Moravian Star with the given dimensions with fluid in order to check the students' volume calculations. If they are incorrect, then the teacher will ask the students to look back at their work and decide where they can make corrections. If they find their math to be correct, then she will ask them to speculate about other possible errors. -The teacher will segue into saying: I would love to show the company pictures of prototypes. So do you think we could make some Moravian Stars this week to the boss of the company? We will choose the best Moravian Star at the end of the week. Each of you can take your own star home and decorate it however you'd like, but this would really help my presentation to the company if I had one prototype to show the company. Whoevers star I use will be returned to them, of course. -Were the specific, detailed measurements that we did today very important? Could we have given *approximate* values, or did we need to be accurate? Why? -How did our investigation inform our design today?

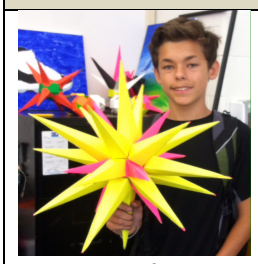
DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	
The content is the same for all students. If a group seems to be more stumped than anticipated, then the teacher will guide that group through finding the surface area only, as the Pythagorean theorem is easier to apply to the lateral face than the height of the entire pyramid.	The process is the same for all of the groups, with the exception that the teacher may probe some groups more than others to remember previous knowledge to apply to this project.	The product may be modified for groups that need more help. A group may only be asked to find the lateral height of a pyramids face, for example. Or they may be asked to only find the surface area of ONE pyramid and not the entire Moravian Star. The teacher will have to gauge this as she asses each group's discussion. In their pairs, the students will type their findings out for the company. This will be needed by the end of class on Thursday.	

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)



Engage and Connect

The teacher will say:

I have a problem. I am working with a company to make a prototype of a Moravian Star (she will have one to display) to eventually make and sell, but they need my exact dimensions by next week. I know some of the dimensions, but in order to make the prototypes, they need to know how much material each star will use. Another issue is that after they make the stars, they are thinking of filling them with a glowing fluid. So I need to tell them how many fluid ounces each star could hold. In your pairs, you need to type your findings out for the company. This will be needed by the end of class on Thursday.

Explore

-The teacher will put the students into groups or three to discuss and draw a plan. The groups will be random, as this is the first day of class, and the ability level and work ethic of each student is unknown.

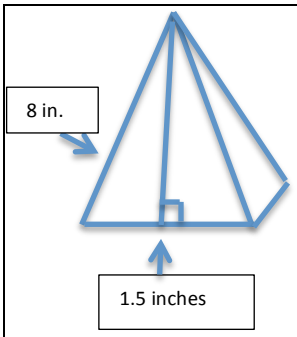
The teacher will say:

Let me show you the parts of the star that I know already, and I'd like us to come up with what other measurements I need in order to give the company what they want.

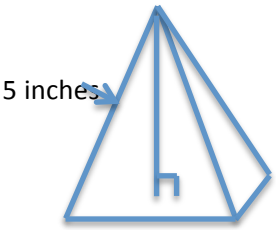
(Hopefully, the students mention the following. The teacher will probe them to mention them, if necessary. They may mention useful things that are not listed below that we can use.)

- Surface Area of Each Pyramid= (sum of lateral faces areas)+(base area)
- Pythagorean Theorem to get the lateral height of each face of each pyramid in order to calculate the surface area.
- Area of a Triangle= $\frac{1}{2}bh$
- Volume of Pyramid= $\frac{1}{3}(base\ area)(height)$
- Area of a square= side*side
- 1 fluid ounce = 1.8 cubic inches
- Pythagorean Theorem to get the height of each pyramid in order to calculate the volume.

Large points: 18 of them (4-sided base)



<u>Small points</u>: 8 of them (3-sided base)



Explain

The students will take turns as groups explaining to the rest of the class how they came up with the information for Miss Parise to give to the company. After each presentation, the other students will have the opportunity to give their insights and opinions about the presenting group's findings.

Elaborate

After every group has presented, the teacher will take the best answers from each group and write them on a final, large paper at the front of the room. This will be the final presentation that she will give to the prototype company.

Evaluate

The teacher will fill a Moravian Star with the given dimensions with fluid in order to check the students' volume calculations. If they are incorrect, they will look back at their work and decide where they can make corrections. If they find their math to be correct, then they will speculate about other possible errors. The teacher will ask the students, "were the specific, detailed measurements that we did today very important? Could we have given *approximate* values, or did we need to be accurate? Why?"

The teacher will say: I know that you going to type a plan for the company, but I would also love to show the company some prototypes. So do you think we could make some Moravian Stars this week to show the company? Each of you can take your own star home and decorate it however you'd like after the company sees them, but these visuals would really help my presentation to the company.

TEACHER NAME			Lesson #
Brooke Parise			2
MODEL	**CONTENT AREA**	**GRADE LEVEL**	
Questioning	Mathematics	8	
CONCEPTUAL LENS		**LESSON TOPIC**	
Investigation		Construction	

LEARNING OBJECTIVES *(from State/Local Curriculum)*

CCSS.Math.Content.HSG.CO.D.12
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

THE ESSENTIAL UNDERSTANDING ***(What is the overarching idea students will understand as a result of this lesson?***	**THE ESSENTIAL QUESTION** ***(What question will be asked to lead students to "uncover" the Essential Understanding)***
Investigation Informs Design	How does Investigation Inform Design?
CONTENT KNOWLEDGE **(What factual information will students learn in this lesson?)**	**PROCESS SKILLS** **(What will students be able to do as a result of this lesson?)**
The students will know how to construct three-dimensional shapes based on reading instructions and using a straight edge, pencil, compass, and scissors.	The students will analyze technical writing and infer how to construct three-dimensional shapes based on those instructions.

GUIDING QUESTIONS

What questions will be asked to support instruction?
Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	**During Lesson Questions:**	**Post Lesson Questions:**
1. How many of you have built something by reading through instructions before? Tell us about that experience: what was difficult? What seemed easy? 2. Who can describe what a square-based pyramid is? Who can describe what a triangular-based pyramid is? Does anyone know what a rhombicuboctahedron is?	The teacher will ask questions about the construction of the base and points of the star ONLY AS NEEDED. If a student is uncertain how to draw the tabs, then she will refer them to the picture on the directions and ask them to explain what THEY think the tabs should look like and how they will go about cutting them out. The teacher will not ask direct questions unless the students need her assistance with the construction.	1. How did investigation inform design with the construction of the base and points today? Could you have constructed the parts of the Moravian Star without investigating the directions? 2. What was easy about what you did today? What seemed to be more difficult?

DIFFERENTIATION ***(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.***		
Content	**Process**	**Product**
Day 1's lesson on using Trigonometry to find the surface area and volume will be the same for all students. On Day 2-4, the teacher will simply give the directions, and let the students work in pairs chosen by her. The pairs are the same pairs from day 1. If a pair of students needs help, she can spend her time guiding both through the part of the project that stumps them.	The students will make their stars and have to measure using compasses and rulers. They will have to use dexterity and logic in order to construct the triangle**. Some students will be able to complete this entire process on their own. Some students will have to be guided through it completely**. The teacher may construct ONE pyramid as an example to some students. She will question them to guide her through every step.	The product will be the same for all students: the Moravian Star prototype and a report to the company with the measurements, surface area, and volume of the project.

PLANNED LEARNING EXPERIENCES
(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Hook: The teacher will show the students a 26-pointed Moravian Star. She will remind the students about making the prototypes for the company in order to get approval to make the stars. After the company sees the stars, then the students will decorate them in any way they like, and they will take them home. (The students will be told that this is a hypothetical company, and that they will be decorating the stars on Thursday to take home on Friday.)

Explore *:*

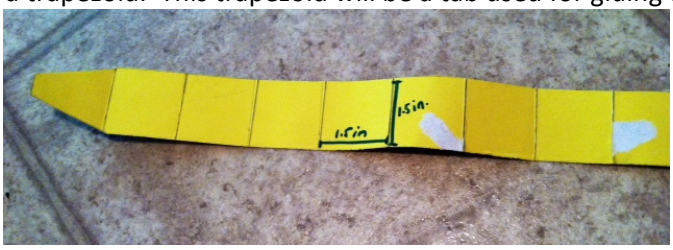
Outline of Project:
Day 1: Calculate the Volume and Surface Area of a 26-pointed Moravian Star. Present to the class your report to company explaining dimensions and volume measurements. The class will choose the best report to present to the manufacturing company.
Day 2: Build a Moravian Star. (The students will probably build the base and begin drawing the points.)
Day 3: Build a Moravian Star. (The students will finish the points and begin attaching them to the base. Some students may finish the star.)
Day 4: Finish building a Moravian Star and decorate it. The students will present the prototypes to the class, and the best will be chosen to present to the company. Take finished product home.

Explain :
The teacher will give the students the Moravian Star instructions. She will visit each pair of students to evaluate whether or not they need her guidance, and how much. Some pairs may build the star independent from her instruction, while others may need step-by-step help. Typically, students will cut the pieces with success, but then they need a demonstration on how to fold and attach the pieces together.

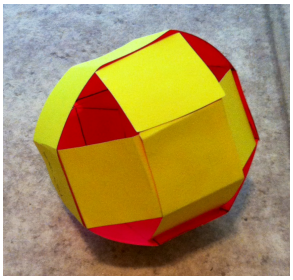
Evaluate:
The final star and final report to the company will be used as an informal evaluation. On Thursday, the groups will present their stars and reports to the other class members. Their peers will be asked to judge whether or not they think the company will approve the design and build it.

Directions:

1. Get a compass, ruler, pencil, 3 pieces of poster board, a glue gun and glue sticks, and spray paint, glitter, or anything with which you would like to decorate your star.
2. Base Construction:
   a. Draw a 13.5 inch by 1.5 inch rectangle on one piece of your poster board. Mark a line every 1.5 inches length wise. Draw this three times total. Cut out the three strips, and form the 9th square into a trapezoid. This trapezoid will be a tab used for gluing the base together.



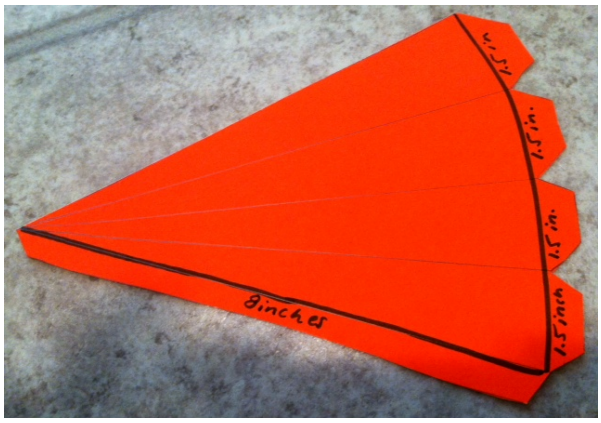
   b. Score the 8 lines with a pair of opened scissors. Fold on each line.
   c. Glue the trapezoidal tab to the square farthest away from it. Do this to all three strips.
   d. After the three strips glue has dried, glue them together as somewhat of a sphere. (This shape is actually a rhombicuboctahedron.) Two of the strips should face longitude direction, and one strip should face latitude direction.



   e. Put this aside for later. The small points will be attached to the inside of the holes, and the large points will be attached to each of the squares.

3. Large point construction

   a. Draw an 8 inch line on the poster board. Hold your compass point at one end of the line and the pencil at the other end of the line. Draw about ¼ of a circle using the compass.
   b. From the 8 inch line, mark every 1.5 inches up the curve. Do this four times. Now, draw a line from the point of the 8 inch line to each of the 1.5 inch markings. This will separate your point into its four lateral sides.
   c. This is very important for gluing purposes: Draw a trapezoid at the top of each 1.5 inch line. (See picture below.) Also, draw a trapezoid after the fourth lateral face. Tip: It is better to draw your trapezoids too big rather than too small. You can always cut away excess paper as needed later, but if the trapezoids are not large enough to hold glue, then you will have to remake points that can hold glue.



d. Score every line. Also score the curve. Bend at each place you have scored. Attach the tab to the lateral face farthest from it. Glue the bottom four tabs together.

e. Repeat this 17 more times. A total of 18 large points is needed.

4. Small point construction.

a. Draw a 5 inch line on the poster board. Hold your compass point at one end of the line and the pencil at the other end of the line. Draw about ¼ of a circle using the compass.

b. From the 5 inch line, mark every 1.5 inches up the curve. Do this three times. Now, draw a line from the point of the 5 inch line to each of the 1.5 inch markings. This will separate your point into its three lateral sides.

c. This is very important for gluing purposes: Draw a trapezoid at the top of each 1.5 inch line. (See picture below.) Also, draw a trapezoid after the fourth lateral face. Tip: It is better to draw your trapezoids too big rather than too small. You can always cut away excess paper as needed later, but if the trapezoids are not large enough to

hold glue, then you will have to remake points that can hold glue.

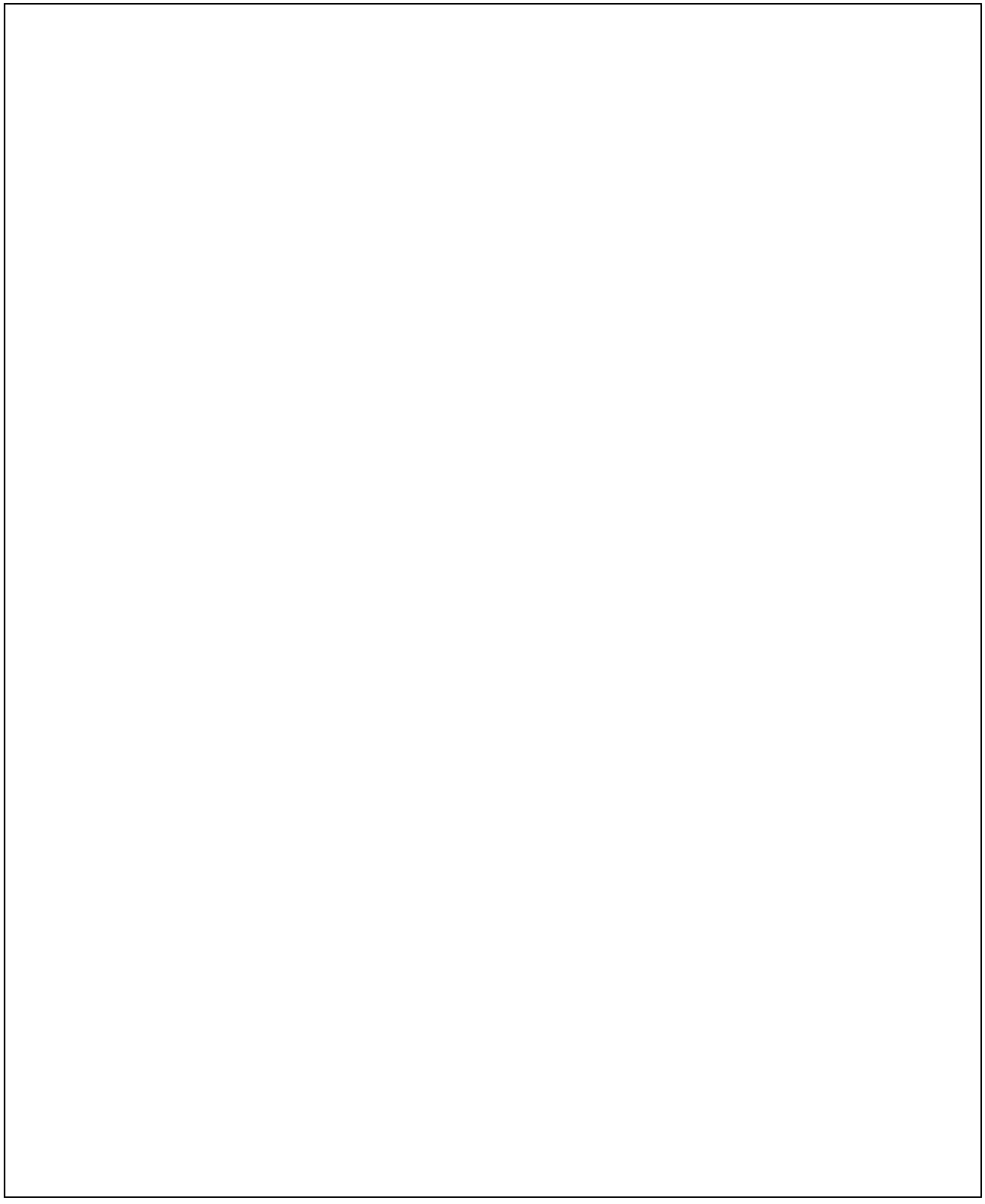
d. Score every line. Also score the curve. Bend at each place you have scored. Attach the tab to the lateral face farthest from it. DO NOT GLUE THE BOTTOM THREE TABS TOGETHER.

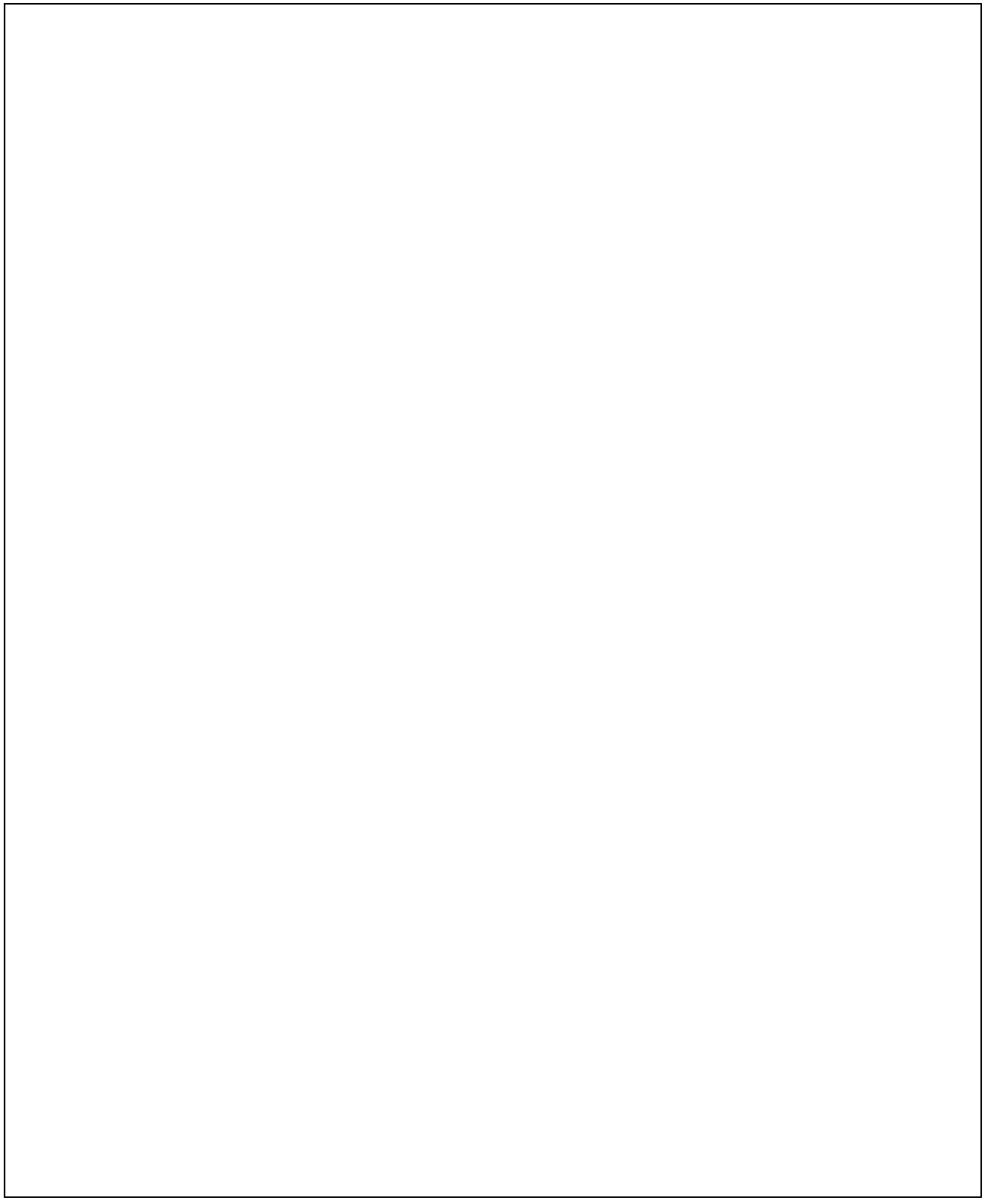
e. Repeat this 7 more times. A total of 8 small points is needed.

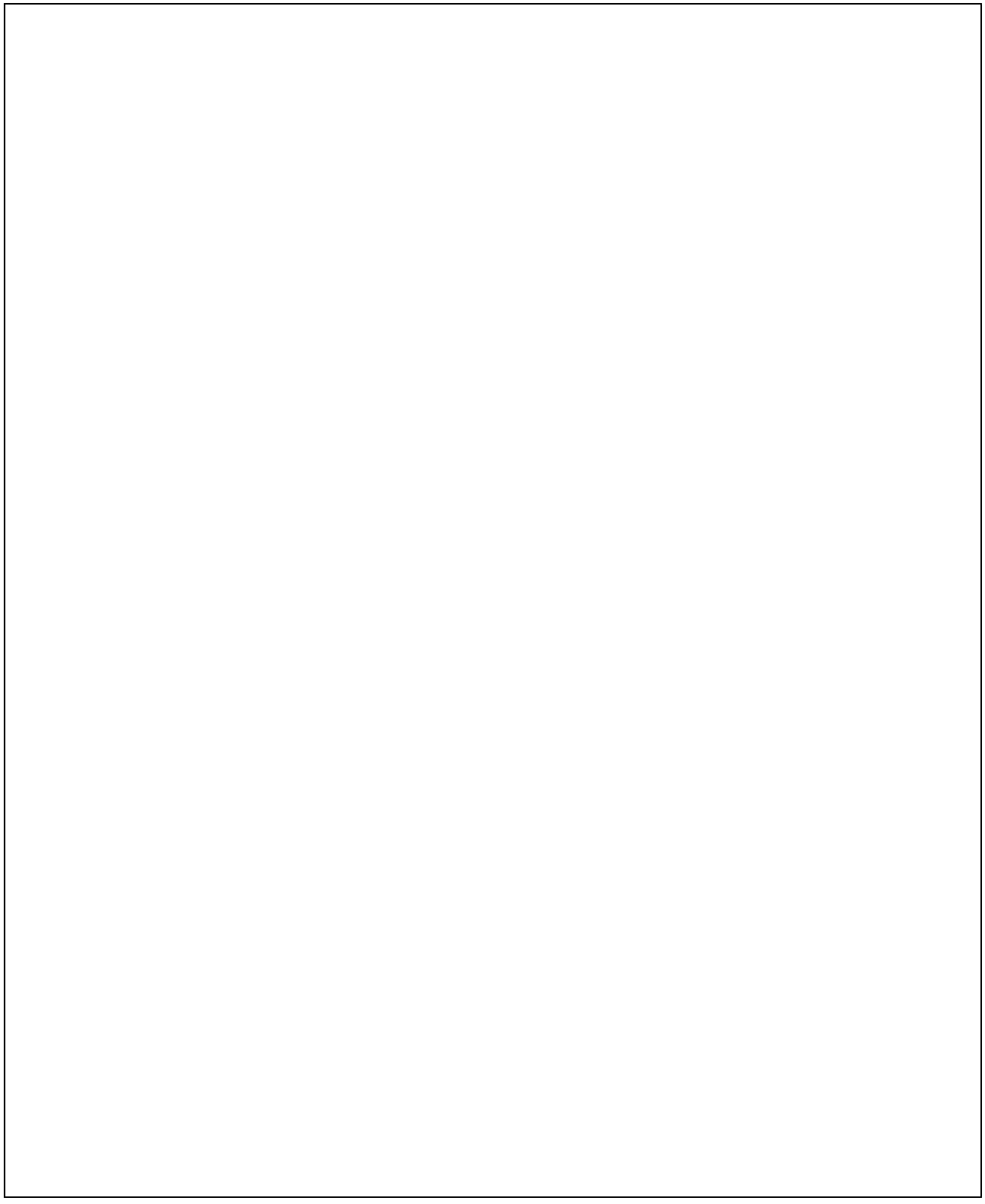
5. Attaching the Points (If you want to attach a fishing line in order to hang the star, then attach it to the base rhombicuboctahedron before attaching the points.

a. Attach the small points first. Each small point's trapezoidal tabs should be glued on the underside, inside of the sphere. You will attach the last small point on the outside of the sphere, as no holes are remaining to reach through.
b. Attach all of the large points to each of the squares.

6. Decorate the star if desired. Because paint drips and is difficult to apply evenly, spray paint is the most useful decorating tool for the Moravian Star.







TEACHER NAME	Lesson #
Brooke Parise	3 (This is the 3rd lesson for STEP summer school.)

MODEL	CONTENT AREA	GRADE LEVEL
Questioning	8th Grade Math	8

CONCEPTUAL LENS	LESSON TOPIC
Investigations	Construction

LEARNING OBJECTIVES *(from State/Local Curriculum)*

CCSS.Math.Content.HSG.CO.D.12
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

THE ESSENTIAL UNDERSTANDING ***(What is the overarching idea students will understand as a result of this lesson?***	THE ESSENTIAL QUESTION ***(What question will be asked to lead students to "uncover" the Essential Understanding)***
Investigation Informs Design	How does investigation inform design?

CONTENT KNOWLEDGE **(What factual information will students learn in this lesson?)**	PROCESS SKILLS **(What will students be able to do as a result of this lesson?)**
The students will know how to construct three-dimensional shapes based on reading instructions and using a straight edge, pencil, compass, and scissors.	The students will analyze technical writing and infer how to construct three-dimensional shapes based on those instructions.

GUIDING QUESTIONS
What questions will be asked to support instruction?
Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	During Lesson Questions:	Post Lesson Questions:
What was something that you did yesterday that was difficult? What seemed easy? Do you have any apprehensions about finishing the construction of the Moravian Star today?	The teacher will ask questions about the construction of the base and points of the star ONLY AS NEEDED. If a student is uncertain how to draw the tabs, then she will refer them to the picture on the directions and ask them to explain what THEY think	How did investigation inform design with the attachment of the base and points today? Do you think that the company is going to understand the construction of the Moravian Star

	the tabs should look like and how they will go about cutting them out. The teacher will not ask direct questions unless the students need her assistance with the construction.	based on my instructions? What would you change about my instructions to make it easier for them?

DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	
On Day 2-4, the teacher will simply give the directions, and let the students work in pairs chosen by her. The pairs are the same pairs from day 1. If a pair of students needs help, she can spend her time guiding both through the part of the project that stumps them.	The students will make their stars and have to measure using compasses and rulers. They will have to use dexterity and logic in order to construct the triangle**. Some students will be able to complete this entire process on their own. Some students will have to be guided through it completely**. The teacher may construct ONE pyramid as an example to some students. She will question them to guide her through every step.	The product will be the same for all students: the Moravian Star prototype and a report to the company with the measurements, surface area, and volume of the project.	

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Hook: The teacher will remind the students about making the prototypes for the company in order to get approval to make the stars. After the company sees the stars, then the students will decorate them in any way they like, and they will take them home. (The students will be told that this is a hypothetical company, and that they will be decorating the stars on Thursday to take home on Friday.)

Outline of Project:

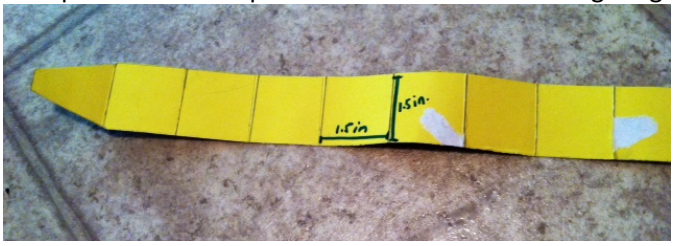
Day 1: Calculate the Volume and Surface Area of a 26-pointed Moravian Star. Present to the class your report to company explaining dimensions and volume measurements. The class will choose the best report to present to the manufacturing company.
Day 2: Build a Moravian Star. (The students will probably build the base and begin drawing the points.)
Day 3: Build a Moravian Star. (The students will finish the points and begin attaching them to the base. Some students may finish the star.)
Day 4: Finish building a Moravian Star and decorate it. The students will present the prototypes to the class, and the best will be chosen to present to the company. Take finished product home.

Explain
The teacher will continue to circulate to each pair to evaluate whether or not they need her guidance, and how much. Some pairs may build the star independent from her instruction, while others may need step-by-step help. Typically, students will cut the pieces with success, but then they need a demonstration on how to fold and attach the pieces together.

Evaluate
The final star and final report to the company will be used as an informal evaluation. On Thursday, the groups will present their stars to the other class members. Their peers will be asked to judge whether or not they think the company will approve the design and build it.

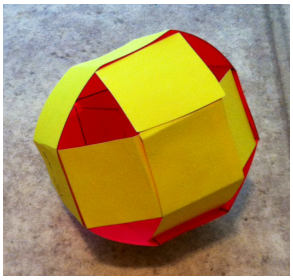
Directions:

1. Get a compass, ruler, pencil, 3 pieces of poster board, a glue gun and glue sticks, and spray paint, glitter, or anything with which you would like to decorate your star.
2. Base Construction:
   a. Draw a 13.5 inch by 1.5 inch rectangle on one piece of your poster board. Mark a line every 1.5 inches length wise. Draw this three times total. Cut out the three strips, and form the 9th square into a trapezoid. This trapezoid will be a tab used for gluing the base together.



   b. Score the 8 lines with a pair of opened scissors. Fold on each line.

c. Glue the trapezoidal tab to the square farthest away from it. Do this to all three strips.
d. After the three strips glue has dried, glue them together as somewhat of a sphere. (This shape is actually a rhombicuboctahedron.) Two of the strips should face longitude direction, and one strip should face latitude direction.



e. Put this aside for later. The small points will be attached to the inside of the holes, and the large points will be attached to each of the squares.

3. Large point construction

a. Draw an 8 inch line on the poster board. Hold your compass point at one end of the line and the pencil at the other end of the line. Draw about ¼ of a circle using the compass.
b. From the 8 inch line, mark every 1.5 inches up the curve. Do this four times. Now, draw a line from the point of the 8 inch line to each of the 1.5 inch markings. This will separate your point into its four lateral sides.
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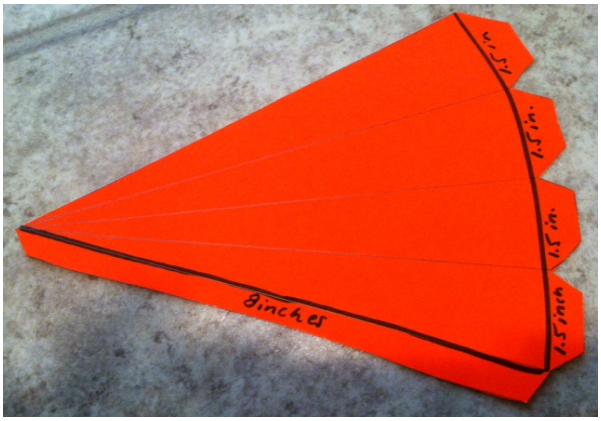

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e. Repeat this 17 more times. A total of 18 large points is needed.

4. Small point construction.

a. Draw a 5 inch line on the poster board. Hold your compass point at one end of the line and the pencil at the other end of the line. Draw about ¼ of a circle using the compass.
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d. Score every line. Also score the curve. Bend at each place you have scored. Attach the tab to the lateral face farthest from it. DO NOT GLUE THE BOTTOM THREE TABS TOGETHER.

e. Repeat this 7 more times. A total of 8 small points is needed.

5. Attaching the Points (If you want to attach a fishing line in order to hang the star, then attach it to the base rhombicuboctahedron before attaching the points.

a. Attach the small points first. Each small point's trapezoidal tabs should be glued on the underside, inside of the sphere. You will attach the last small point on the outside of the sphere, as no holes are remaining to reach through.
b. Attach all of the large points to each of the squares.

6. Decorate the star if desired. Because paint drips and is difficult to apply evenly, spray paint is the most useful decorating tool for the Moravian Star.

TEACHER NAME		Lesson #
Brooke Parise		4 (This is the 4th lesson for STEP summer school.)
MODEL	**CONTENT AREA**	**GRADE LEVEL**
Questioning	8th Grade Math	8

CONCEPTUAL LENS	LESSON TOPIC
Investigations	Construction and Presentation

LEARNING OBJECTIVES *(from State/Local Curriculum)*

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real world and mathematical problems.

CCSS.Math.Content.HSG.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	THE ESSENTIAL QUESTION *(What question will be asked to lead students to "uncover" the Essential Understanding)*
Investigation Informs Design	How does investigation inform design?

CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	PROCESS SKILLS (What will students be able to do as a result of this lesson?)
The students will know: - Surface Area of A Pyramid= (sum of lateral faces' areas)+ (base area) - Pythagorean Theorem : $a^2 + b^2 = c^2$ - Area of a Triangle $=\frac{1}{2}(base)(heigt)$ - Volume of Pyramid = $=\frac{1}{3}(base\ area)(height)$ - Area of a square = (length)(width) - 1 fluid ounce = 1.8 cubic inches The students will know how to construct three-dimensional shapes based on reading instructions	The students will analyze the Moravian star and break it down into its individual parts in order to find the surface area and volume of the entire structure. They will present their findings in the form of a report to be sent to a manufacturing company. The students will convert their volume units, which are $inches^3$, to ounces. The students will analyze technical writing and infer how to construct three-dimensional shapes based on those instructions.

and using a straight edge, pencil, compass, and scissors.	

GUIDING QUESTIONS

What questions will be asked to support instruction?

Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	During Lesson Questions:	Post Lesson Questions:
<u>The teacher will say</u>: I am eager to hear how you have solved my problem. Do any of you feel that your presentation is ready for the company and WILL be approved for manufacturing? Do any of you feel that you may not have enough information, or do not have accurate information for the company to approve the making of the stars? Why; what do you need in order for the project to be approved?	-The teacher will ask each pair questions based on their personal presentation of the report and the Moravian Star prototype.	- After every group has presented, the teacher will ask the students what they think the best presentation was. Why was that the best presentation? -What was difficult about this investigation? What was easy? In what ways did we have to investigate in order to inform our design?

DIFFERENTIATION ***(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.***			
Content	**Process**	**Product**	
The content is the same for all students. If a group seems to be more stumped than anticipated, then the teacher will guide that group through finding what they could have improved upon in their report.	The process is the same for all of the groups: they are presenting their findings.	The product is the same for all of the groups. The teacher may have helped some groups more than others based on their need, but the teacher's expectation of the final report and Moravian Star is the same for all students.	

PLANNED LEARNING EXPERIENCES
(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect

The teacher will say:
I'm excited to hear how you have helped me with my problem. Today you will be presenting your reports for the company to make a prototype of a Moravian Star. If the company likes your report and prototypes, then they may eventually make and sell my Moravian Star design.

Explore

-Each pair of students will present their Moravian Star prototype to the class. The prototype should be accurately measured and constructed based on the instructions provided. The students will thoughtfully investigate each other's prototype to choose the best among the class.

Explain

After each pair shows their prototype, the other students and the teacher will ask them questions about it. Each pair will answer the questions as if they are presenting to the boss of the company, as this is a business presentation.

Elaborate

The teacher has already chosen the best report on day 1. Today, the teacher and class members will choose the best prototype. This will be the final Moravian Star prototype that she will give to the manufacturing company.

Evaluate

Whether or not the company decides to build the prototype is the final evaluation. One pair will be chosen to present the prototype and report to a special guest, who will be thought of as the "C.E.O. of the company." If the C.E.O. chooses to build the prototype based on the report and model, then the class has succeeded above and beyond expectation. Simply learning to investigate and problem solve were the goals of this project.