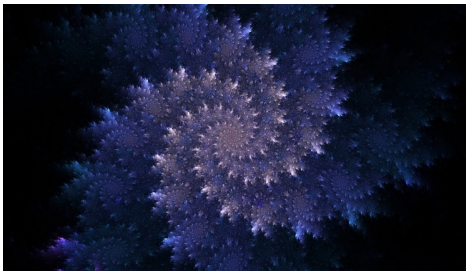
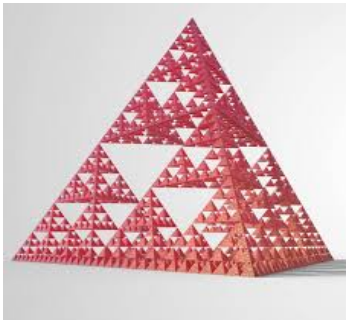
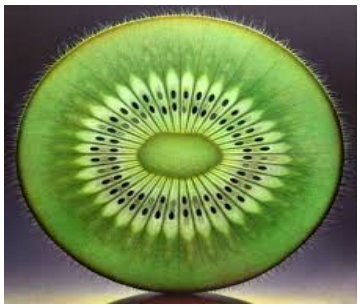


The Mystery of Patterns





Developed By:
Kim Sholler
Designed for Grades 6, 7, 8
July 2020

I Introduction

A. Rationale

This unit focuses on the essential understanding that “patterns define relationships”. This understanding not only applies to mathematical and scientific concepts, it also applies to so many aspects of our daily lives. It is critically important that students learn how to identify patterns (from decision making patterns, to patterns identified in a math or science problem) and understand how they define relationships. The content and skills involved in this unit will help students think differently about the importance of identifying patterns and how these patterns define relationships. Students will be introduced to the concept of Patterns by exploring Fractals. Fractals help us study and understand important mathematical and scientific concepts, such as the way bacteria grow, patterns in freezing water (snowflakes) and brain waves. Think about wireless cell phones. The built-in antennas on wireless cell phones use a fractal pattern to pick up the signals better and pick up a wider range of signals, rather than a simple antenna. Through this investigation, students will develop an understanding of patterns and how they define relationships.

B. Differentiation for Gifted Learners

Dimensions of Differentiation: The curriculum is designed specifically for gifted learners. This unit contains thought provoking videos that feature advanced content differentiation, beyond the scope of standard knowledge and understanding of a middle school math student. As students progress through the lessons in this unit, they will engage in a variety of challenging process skills, starting with a Team-Based Learning activity that requires students to analyze text, synthesize information, create generalizations and provide evidence to support a claim. Then they will use the Questioning model to examine the Fibonacci sequence to examine patterns and relationships before moving on to a lesson based on the Taba model of sorting, naming, regrouping, subsuming and generalizing their ideas. Finally, students will conclude the unit with a Visual Thinking Strategies lesson. This lesson is an open-ended thinking and inquiry strategy which allows students to share unique perspectives and make inferences. Students will articulate meaning they make from visual images during this student - led lesson.

Comment [AG1]: What about complexity, acceleration, creativity, etc? See checklist for final unit.

II Unit Goals and Outcomes

Unit Topic:	**Unit Rates**
Unit Concept:	**Patterns**
Essential Understanding:	**Patterns Define Relationships**
CONTENT Goal and Objectives:	**GOAL:** To develop a visual understanding of unit rates by exploring a two-dimensional tiling pattern with a finite border. **OBJECTIVES:** *The students will know that…* • Exploring unit rates using tiling patterns enables them to see the relationship between different shapes that repeat in continuous and predictable ways. • The table of values will be different depending on the unit tile they identify, but the totals of the values will be the same because all students are working on the same image.
PROCESS Goal and Objectives:	**GOAL:** To understand that ratios can be expressed as equivalent unit ratios by identifying patterns and interpreting both unit ratios in context. **OBJECTIVES:** *The students will be able to…* • To identify patterns and to use ratio language to describe a ratio relationship between two quantities • Use ratio reasoning to solve real-world and mathematical problems • Use ratio reasoning to determine the best deal in relationship to a unit rate
CONCEPT Goal and Objectives:	**GOAL:** To understand the concept of patterns **OBJECTIVES:** *The students will understand that…* • A ratio describes a multiplicative relationship between two quantities • Identifying patterns to find equivalent ratios can be represented in a variety of ways (ratio table, graph, double number line) • Scaling up and scaling down a ratio maintains equivalency

III Assessment Plan

A. Formative Assessment

Formative assessment strategies are embedded throughout this unit.

Assessments embedded in Lesson 1:

- 10 question multiple-choice quiz based on the reading of “Mr. Max the Math Teacher” This quiz is used to help students see that there are patterns in decision making that define the relationships we have with a given topic.
- During student discussions, the teacher will observe and listen to student conversations to see if they are making connections related to patterns in decision-making.
- Lastly, the students will either write a reflection or create a video or skit demonstrating what they learned and how the patterns and relationships in their lives inform their decision-making.

Assessments embedded in Lesson 2:

- After watching a short video, students will work to complete a table to show a male bee at generations zero and determine how many ancestors he had in each previous generation one through five. The teacher will make observational notes about students’ understanding.
- Teacher will make notes about student dialogues as they discuss the patterns identified in the bee generation table.
- Students will be given a series of pictures to view. Students will work in groups to use the knowledge they gained in the first part of the lesson to identify the first twelve Fibonacci numbers. They will use this information to calculate the ratio of each number to the previous one. The teacher will make observational notes during this process.
- After watching a short video about DaVinci and the presence of the golden ratio throughout the human body, students will work to find evidence of the golden ratio by measuring limbs. The teacher will monitor understanding by observing and listening to student discussions.
- Students will be asked to compare the results of their investigation with a partner and to discuss what they have learned about patterns and relationships that would help they solve real world math problems. The teacher will make observational notes during the discussion.

Assessments embedded in Lesson 3:

- The teacher will observe as students group and label characteristics of fractal images. During the subsuming, regrouping, and renaming portion of the lesson, the teacher will challenge the students to regroup items.
- The teacher will move throughout the classroom checking in and monitoring student groups as they work together to write a generalization that is always true about patterns.
- Student groups present their generalization then the whole group votes on the generalization that best supports the essential understanding that patterns define relationships.

Assessments embedded in Lesson 4:

- The teacher will observe as students create a fractal image. The teacher prompts the students to think about the patterns in their image and where else these patterns may be found in math, nature, architecture, etc.
- The teacher observes, as students will work in small groups to analyze an image. Each small group will use VTS strategies.
    - **Questions:**
        - What do you see?
        - What do you see that makes you say that?
        - What else do you see?
- Student groups presents their image and the following information:
        - The name of the image or some identification for the image.
        - Patterns represented in the image
        - How do the patterns represented in the image create a fractal image?
    - As student groups present, students listen and take notes on a graphic organizer.
- Students are given the following academic prompt: How do patterns define relationships?

B. Summative Assessment

The following performance task will be presented to students as a summative assessment.

Performance Task

In light of the recent Covid 19 Pandemic, the Town of Durham has decided to create a community garden to help provide fresh vegetables for community members in need of food.

You are a landscaper for the town of Durham and have been asked to design a garden using a 500 square foot empty plot of land near the town center.

You will need to build a fence around the garden to protect it from animals. Your garden must have two four-foot gates. You may want to try different shapes that can be made within your 500 sq. foot parcel of land and calculate the areas to compare. As you design your garden for maximum growing space, you may want to consider how the patterns in your table determine the relationship between length and width of the sides and the total area of growing space.

You will need to present your design plans and price estimates to the director of the planning board for approval before executing your plan. Your presentation should include at least four garden configurations, with scale drawings of the configurations. You will also need to include your area configurations in a table for comparison purposes. In addition, you will need to provide a cost estimate from at least two local home improvement stores. You must include the price per square foot for comparison purposes. Finally, you will need to write an executive summary of your recommendations justifying the design choice that best meets the criteria.

Performance Task Rubric

	Level 1	Level 2	Level 3	Level 4
Problem Solving Approach	Approach is random; only one solution found.	Approach is not very systematic. Several solutions are compared.	Approach is systematic. Multiple solutions are compared.	Approach is highly systematic and reveals comprehensive understanding. Multiple solutions are compared and the best solution is identified.
Accuracy and Procedural Skill	Many computational errors. Incorrect formulas are used for calculations. Structure of the problem is not understood.	A number of computational errors. Or Incorrect Formulas are used for calculations. There is a developing level of understanding of the structure of problem.	Very few computational errors. Correct formulas are used for calculations. Students understand the structure of the problem.	Solution is accurate, and the calculations demonstrate understanding of the structure of the problem.
Communication	Drawing is unclear and erred. There is no labeling. Students demonstrate little to no recognition of the effect of changing the number and lengths of the sides on the area of a polygon. No explanation is written. Students are not able to explain how patterns define relationships.	Drawing is mostly clear; some errors in scale and / or labeling. There is some recognition of the effect of changing the number and lengths of sides on the area of a polygon. Explanation demonstrates a developing level of understanding. Students are beginning to make the connection between patterns and relationships.	Drawing is labeled with the correct scale. The effect of changing the number of sides and length of sides on the area of a polygon is mostly understood. Explanation is clearly written. Students are able to explain how patterns define relationships.	Drawing is labeled correctly and is well represented revealing understanding of the relationship between number of sides, lengths of sides, and area of a polygon. Explanation demonstrates mastery of the concept and student is able to explain how patterns define relationships.

Comment [AG2]: Review the descriptors for consistency in measuring /assessing product.

IV Lesson Plans

TEACHER NAME			Lesson #
Kimberly Sholler			1
MODEL	**CONTENT AREA**	**GRADE LEVEL**	
Team Based Learning	Math/ELA	Grades 6	

CONCEPTUAL LENS	LESSON TOPIC
Patterns	Decision-Making

LEARNING OBJECTIVES *(from State/Local Curriculum)*

- Literacy CCRA.R1: Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
- Literacy CCRA.R2: Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
- Literacy CCRA.R10: Read and comprehend complex literary and informational texts independently and proficiently.
- 6.RP.3: Students will use ratio and rate reasoning to identify patterns and to solve real-world mathematical problems by reasoning about tables of equivalent ratios or equations.

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	**THE ESSENTIAL QUESTION** *(What question will be asked to lead students to "uncover" the Essential Understanding)*
Patterns define relationships	How do patterns define relationships?
CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	**PROCESS SKILLS** (What will students be able to do as a result of this lesson?)

Students will know that:	Students will be able to:
• Math is everywhere • Math concepts can be applied in everyday activities not just when sitting in math class • Understanding practical applications of math helps make math more meaningful • Patterns in our academic decision making define relationships and connections to the subject • Connecting text to personal experiences allows us to establish a deeper understanding of the passage • Decision making patterns define our relationship with the topic	• Analyze text • Synthesize information • Create generalizations • Provide evidence to support a claim • Work collaboratively • Come to a consensus • Establish and defend a position • Integrate Information • Manipulate information

GUIDING QUESTIONS

What questions will be asked to support instruction?

Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	During Lesson Questions:	Post Lesson Questions:

• What patterns connect math to sports? • What relationships in sports are defined by patterns? • How do patterns and relationships inform the decision-making of musicians and composers? • How do patterns and relationships inform decision-making in business? • What relationships in architecture and construction are defined by patterns? • How do patterns, relationships and connections help you make reading more meaningful? • How do the patterns, relationships and connections inform your decision-making? • How does understanding how many professions require math change your perception of its importance? • How does understanding how many professions require math change your perception of its importance? • How do patterns, relationships and connections help you make math more meaningful?	• What did you learn from the article you read? See 10 question quiz attached for during lesion questions. **Debrief:** • What is your answer to question one? Why did you select that answer? • What is your answer to question two? Why did you select that answer? • What is your answer to question three? Why did you select that answer? • What is your answer to question four? Why did you select that answer? (Continue through question 10) • How does the pattern of Bobby's decision making define his relationship with math? • How can reading help you understand relationships and connections with decisions you have made? • How can reading a passage help you understand personal decisions? • How can reading a passage help you see and understand things from a different perspective? • What patterns can you identify in the decisions you make? • What patterns can you identify in the text? • What specific examples from the text can you relate to? • What personal life experiences are you able to relate to the passage? • How do patterns and relationships inform your decision-making? • What is the value in understanding patterns as they relate to your decision-making process? • What further relationships can you draw from the text? **(After whole class debrief)** • Which questions were challenging for you? • Why do you think they were challenging? • What answers would you challenge? • What is your evidence?	• What patterns did you record? • What further relationships can you draw from the text? • How can you compose a word problem that incorporates your understanding of patterns and how they define relationships? • How can you compose a word problem that incorporates your understanding of patterns and relationships? • What pattern did you see emerge from the video? • How do patterns and relationships influence the decisions you make? • What relationships develop as patterns in decision-making start to emerge? • What did you learn from this lesson? • How can you apply what you learned to future learning opportunities? • How do patterns define relationships? Personally? Academically? Socially? • How do patterns define relationships? • How do patterns and relationships inform decision-making?

DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	Learning Environment
Content is above level.	Students will engage in in-depth critical thinking. Students will respond to open-ended high level questions to reach consensus.		Students will be strategically grouped based on ability and Lexile level. Students will work in collaborative groups as well as in whole groups.

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect - *This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.*

Students will watch two short videos "Investing in our Youth: Math is Everywhere" https://youtu.be/Hh1M4O9ed1
And "Mathematics in real life" https://youtu.be/dpv06SFHtRg

After Students watch the videos, the teachers asks pre-lesson questions:

Pre-Lesson Questions:

- What patterns connect math to sports?
- What relationships in sports are defined by patterns?
- How do patterns and relationships inform the decision-making of musicians and composers?
- How do patterns and relationships inform decision-making in business?
- What relationships in architecture and construction are defined by patterns?
- How do patterns, relationships and connections help you make reading more meaningful?
- How do the patters, relationships and connections inform your decision-making?
- How does understanding how many professions require math change your perception of its importance?
- How does understanding how many professions require math change your perception of its importance?
- How do patterns, relationships and connections help you make math more meaningful?

After watching the videos and discussing the pre-lesson questions, students will be placed in teams of 3 or 4 to further discuss the connections between, math, patterns, relationships and real world applications. Students will develop and create a working definition of patterns and relationships and the importance of understanding the connection to real world applications. If necessary, students can use the internet to research; however, the definition ultimately needs to be created by group members.

Next, teams will be given a piece of chart paper to write and illustrate the definition their team created. All groups will post their chart paper on the wall and teams will do a gallery walk to explore and examine definitions other groups created. Then we will have a class discussion and use the definitions from all groups to make a class definition. At the end of the lesson, we will revisit the definition and see if modifications need to be made based on the learning.

Explore - *In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.*

Step One: iRAT (Individual Readiness Assurance Test)

The teacher provides students with an article to read independently. *Mr. Max the Math Teacher* **(see attached)**

After reading the article, students are provided a ten-question quiz. They are instructed to respond to the quiz independently.

Step Two: tRAT (Team Readiness Assurance Test)

Students are asked to meet with their team. (These teams of 3 or 4 have been pre-determined by the teacher and are static for a period of time. For older students, teams could remain the same for a marking period, whereas younger students may stay in their teams for a shorter period of time.)

In teams, students are instructed to share responses to the ten-question quiz. Students are to come to consensus about the best responses. When students have come to consensus about best responses, they are provided a scratch-off card to confirm best responses. Students must have 100% of responses correct. (if scratch-off cards are not available, the teacher may check responses for each team.) All responses must be correct before teams move to step three.

Explain - *Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.*

Step Three: iFAT (Immediate Feedback Assurance Test)

When all teams have 100% correct responses, the teacher reconvenes the whole group. The teacher debriefs the quiz with the whole group.

Debrief:

- What is your answer to question one? Why did you select that answer?
- What is your answer to question two? Why did you select that answer?
- What is your answer to question three? Why did you select that answer?
- What is your answer to question four? Why did you select that answer?
- (Continue through question 10)

The teacher may ask:

- Which questions were challenging for you?
- Why do you think they were challenging?
- What answers would you challenge?
- What is your evidence?

Students discuss how they determined best responses, why the response was the best response; identify questions, which were challenging and why they were challenging. Students may challenge best responses. When a challenge is issued, students have the opportunity to provide evidence from the text to support the challenge.

<u>During Lesson Questions:</u>

- How does the pattern of Bobby's decision making define his relationship with math?
- How can reading help you understand relationships and connections with decisions you have made?
- How can reading a passage help you understand the patterns of your personal decisions?
- How can reading a passage help you see and understand things from a different perspective?
- What patterns can you identify in the decisions you make?
- What patterns can you identify in the text?
- What specific examples of decision-making patterns from the text can you relate to?
- What patterns in your personal life experiences are you able to relate to the passage?
- How do patterns and relationships inform your decision-making?
- How can you compose a word problem that incorporates your understanding of patterns and how they define relationships?
- What is the value in understanding patterns as they relate to your decision making process?
- What further patterns and relationships can you draw from the text?

Elaborate —*Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways*

Step Four – Mini Lesson
The teacher introduces the video clip below. The teacher instructs students to observe and makes notes of things that involve patterns and how are patterns and relationships connected to decision-making.
Music and math: The genius of Beethoven - Natalya St. Clair https://youtu.be/zAxT0mRGuoY

Then teacher asks:
What patterns did you record?
How are the patterns you recorded are connected to relationships and decision-making?
The teacher solicits multiple responses from students orally.

The teacher asks post lesson questions:

- What pattern did you see emerge from the video?
- How do patterns and relationships influence your decisions?
- What relationships develop as patterns in decision-making start to emerge?
- What did you learn from this lesson?
- How can you apply what you learned to future learning opportunities?
- How do patterns define relationships? Personally? Academically? Socially?

Evaluate: *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*
The teacher asks:

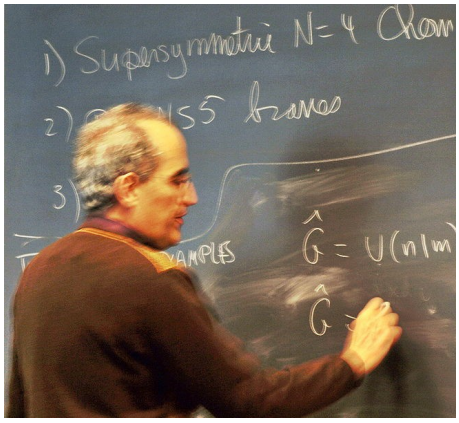
- How do patterns and relationships inform decision-making?
- How do patterns define relationships?

The teacher will ask student reflect on the above questions and either compose an essay or create a video or skit demonstrating what they learned and how the patterns and relationships in their lives inform their decision-making. If students choose to create a video or skit, classmates will be asked to try to identify the patterns that defined relationships in the skit or video before the creators reveal that information.

ReadWorks Article:

Mr. Max the Math Teacher

By Michael Stahl



When Bobby Richardson graduated from elementary school, he left the 5th grade as the best mathematics student in his class. Bobby had the highest math average of anyone in his grade at the end of every year he was in elementary school. At the graduation ceremony, his principal gave him the Math Award after Bobby had earned a 98.7% average on his math tests that year. His teachers, parents and friends were very proud of him. Bobby was proud of himself, too, because he knew how much hard work he put into getting such high grades. Bobby studied math every day and spent extra time on his math homework. He practiced it any time he had a chance. If his mother needed help cooking, Bobby would measure out the ingredients. When his father drove him somewhere, Bobby figured out the distance they'd traveled in miles, yards, feet and inches. Whenever his little sister needed help with her 3rd grade math homework, Bobby was there to tutor her. He did his fair share of work in all other subjects, too – reading books, keeping up with science and studying history – but he and everyone else knew that Bobby Richardson was truly a math whiz.

After graduation, Bobby enjoyed his two-month summer vacation. He went to the Olympic-sized pool quite a bit to swim some laps, figuring out how many meters he'd swum and his average times. He would also play a lot of basketball, adding up all his points and assists, and comparing his stats to those of professional players. Like most kids on summer vacation, even though he was practicing his math, Bobby wasn't thinking too often about school. He did those

calculations because he enjoyed it. Before long, though, the new school year would be approaching.

A week before September arrived, Bobby received his first junior high school class schedule, which was emailed to him by the school. Right away he looked to see when he would be in math class, what level and with what teacher. Sure enough, he had earned his way into the honors class, which would be held every school day during 3rd period. His math teacher's name was Mr. Max. Bobby liked the name; it was kind of funny to have what is usually a first name as a last name. Bobby also imagined that it was short for something silly, like "Maximum Effort!"

Bobby and his family gathered around the dinner table that night. His parents expressed that they were happy to find out that in 4th grade, Lindsay, Bobby's little sister, would learn under the same teacher, Mrs. Snedecker, who taught Bobby two years earlier. Bobby's older brother, Tyler, was entering his senior and final year of high school. Tyler had heard all good things about the teachers he would meet the next week. Finally, Bobby shared his schedule with the table. His parents were happy to hear that Bobby would be in 6th grade honors math even though it was the least surprising thing they'd heard in quite some time. Bobby said his teacher's name was Mr. Max. Tyler, who had also attended Broderick Junior High, gave a responsive howl and said, "Mr. Max, eh?"

"Yeah, why?" Bobby asked anxiously and curiously.

"He was my sixth grade math teacher, too," replied Tyler, who then patted Bobby on the back of the shoulder. "He's, um, interesting," Tyler added.

Bobby kind of shuddered. He certainly was confident that he would perform well in math class just as he always did, but he was not sure whether it would be as easy as it had been during his elementary school days.

During his 6th grade honors math class on the first day of junior high, Bobby met Mr. Max. He didn't seem strange to Bobby at all. Mr. Max was about 6 feet tall, looked physically fit and wore nice clothes with a tie. He greeted the class with a warm "Good morning!" and went over the syllabus and his expectations for the year, none of which seemed terribly unachievable.

"Just wait," warned Tyler that night with a grin and a nod of the head. "You'll see that Mr. Max is different."

Day two of math class with Mr. Max also felt normal to Bobby. Mr. Max began to teach them a little bit of algebra. For the first time in his life, Bobby saw letters in math problems. When Bobby got home that day, he did his homework and reviewed his notes from the day. He said to Tyler, “I still don’t understand why you said that stuff about Mr. Max.”

All Tyler did was shake his head and smile again, saying, “Wait.”

The next day, at the beginning of math class, Mr. Max walked in with a rocking chair, placing it in the front of the room, next to his huge desk. He sat down and said, “Listen closely to everything I say. Feel free to take notes on some paper.” Bobby knew this was the moment Tyler had previewed for him, but at the same time, he tried to forget it and do what Mr. Max had instructed.

Mr. Max sat in his rocking chair and began to rock forward and backward. He told the story about himself when he was a boy. He discussed his relationships with his parents and his sister. He talked about his town, what it looked like and some of the people he knew there. Mr. Max then went over what foods he liked to eat and a bunch of other things about what he did back then. And that was pretty much it. All of the students in the class looked around in confusion. None of what Mr. Max said seemed to have anything to do with math. Mr. Max finished by saying that the class would have their first quiz the next day, and it would be on what they had learned.

Like the rest of his class, Bobby was anxious about his first quiz. Mr. Max smiled as he gave out the sheets of paper with the questions written on them. Sure enough, the quiz had questions about Mr. Max’s boyhood, but they didn’t appear to be answerable based on his speech. The first question was about his favorite food, apples, and read: *If I ate 57 apples when I was sixteen and know the total amount of apples I ate when I was sixteen and seventeen is 134, how many apples did I eat when I was seventeen?* All of the other questions, five in total, were similarly about the number of relatives he had, places he had been and sports games he attended. Bobby did his best, but he was not his usual confident self.

The next day was Friday. Bobby and all his schoolmates were tired from their first week of school, but most of them were jittery about the grades on their first math test. To their surprise, Mr. Max gave the tests back, but there were no grades written on them. Mr. Max went to the front of the room and began to go over all the questions. Suddenly, it became clear to Bobby what Mr. Max was up to. Mr. Max did not write one word on the blackboard.

Instead he wrote algebraic equations just like those they had gone over on Tuesday. Question number one turned into *x+57=134* and so on and so forth.

After he was finished going over all the answers, Mr. Max said, “It’s time for your second quiz.” The whole class groaned. “It’s a quick one,” Mr. Max insisted. “But this one actually counts. So get serious.”

Mr. Max handed out little sheets of paper. When Bobby got his, he read one, single question that he had to answer: *What have you learned today?*

Bobby thought for a little while, remembering all the times he used math while cooking with his mother and driving with his father, or when he went swimming and played basketball.

Then, he wrote: *Math is everywhere.*

The next week, Mr. Max would tell Bobby that he earned a perfect score of 100% on his first junior high school math quiz, as did the rest of his class, who were all relieved. For the rest of the year, they would give Mr. Max maximum effort because they never knew what surprises would be in store for them.

TBL Quiz: Mr. Max the Math Teacher

Independently respond to each of the questions below by circling the letter that represents the ***BEST*** answer.

1. Which statement below best describes Bobby's relationship with math?
   A. Bobby saw math everywhere and enjoyed it.
   B. Bobby viewed math as just another subject in school.
   C. Bobby did not really like math, but he did it because he knew it was important.
   D. Bobby viewed math as a chore math.

2. Why did Mr. Max tell stories about his family, his childhood experiences and the relationships he had with his parents and sister during Math class?
   A. He didn't feel like teaching that day.
   B. He thought the students would be amused by his stories.
   C. He wanted to use his experiences to connect math with relationships.
   D. He wanted to prepare students for their first quiz about his life.

3. Why do you think Mr. Max did not grade the first quiz he gave to his 6^{th} grade class?
   A. He was just lazy.
   B. He did not think any of the students would be able to pass the quiz.
   C. He wanted to introduce his students to a different way to connect with math.
   D. He wanted to make sure his students liked him.

4. How does the pattern of Bobby's decision making define his relationship with math?
   A. It does not have any effect on it at all.
   B. Bobby saw math in everyday life and loved practicing math any chance he got.
   C. Bobby was just naturally good at math that is why he likes it so much.
   D. Bobby saw math as a chore, but he knew if he practiced a lot, he would be good at it.

5. What is the pattern that helped you identify the theme of this story?
   A. Having too much confidence in yourself can be harmful.
   B. Trusting the judgement of others is never a good idea
   C. Competing with others can make you better at something.
   D. Math can be found and used in everyday activities and connects patterns and relationships to decision-making.

6. How should Bobby's past decision-making patterns influence the way his brother tries to make Bobby feel about math when he describes Mr. Max?
   A. Bobby should feel apprehensive.
   B. Bobby should feel proud.
   C. Bobby should feel exhausted.
   D. Bobby should feel confident.

7. How did the result of Bobby's decision-making patterns help identify the resolution of the story?
   A. Bobby's confidence in his ability and the pattern of his decisions prepared him to earn a perfect score on his first junior high school math quiz.
   B. Bobby's performance in 5^{th} grade helped him get a perfect score on his first junior high school math quiz.
   C. Bobby was well prepared for junior high school and was excited to receive his schedule by email.
   D. Mr. Max brings a rocking chair to class and talks about his childhood and gave all of his students a perfect score on their first junior high school math quiz.

8. What patterns developed as students interacted with Mr. Max? Choose the best two answers.
   A. The students became more uncertain of Mr. Max's expectations.
   B. The students realized that Mr. Max wanted to develop an authentic relationship with them.
   C. The students realized that Mr. Max just liked talking about his family.
   D. The students realized that establishing a relationship with Mr. Max informed their patterns in decision-making.

9. What patterns and relationships did Bobby demonstrate that help the reader understand that Bobby believes math is everywhere?
   A. Bobby just likes math so he looks for it.
   B. Bobby is able to see the relationship between math and his every day activities.
   C. Bobby is a geek.
   D. Bobby wants to make sure he is the smartest one in his family.

10. Which of these statements identify the patterns in Bobby's decision-making?
   A. He calculates how many meters he swims and averages his times.
   B. Bobby uses measuring skills in the kitchen when he helps his mom cook.
   C. When Bobby is driving with his dad, he calculates the distance traveled in miles, yards, feet and inches.
   D. Bobby measures surfaces in his house to calculate area and perimeter.

TEACHER NAME			Lesson #
Kimberly Sholler			2
MODEL	**CONTENT AREA**	**GRADE LEVEL**	
Questioning	Math	6	
CONCEPTUAL LENS		**LESSON TOPIC**	
Patterns		Ratios	

LEARNING OBJECTIVES *(from State/Local Curriculum)*

6.RP.1: Students will understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.3: Students will use ratio and rate reasoning to identify patterns and to solve real-world mathematical problems by reasoning about tables of equivalent ratios or equations.

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	**THE ESSENTIAL QUESTION** *(What question will be asked to lead students to "uncover" the Essential Understanding)*
Patterns define relationships	How do patterns define relationships?

CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	**PROCESS SKILLS** (What will students be able to do as a result of this lesson?)
Students will know that: • A ratio is a comparison of two numbers • Multiplying both parts of a ratio by the same number generates equivalent ratios • The ratio of Fibonacci Numbers can be used to derive the golden ratio • Patterns can be used to generate equivalent ratios • Proportional relationships can be used to solve multi-step real world math problems by identifying patterns, relationships and ratios	Students will be able to: • Analyze text • Synthesize information • Create generalizations • Provide evidence to support a claim • Work collaboratively • Come to a consensus • Establish and defend a position • Integrate Information • Manipulate information

GUIDING QUESTIONS

What questions will be asked to support instruction?

Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	**During Lesson Questions:**	**Post Lesson Questions:**

• What is a pattern? • Where are places we find or see patterns? • What are some characteristics of patterns? • What might patterns illustrate? • What do you think about when I say "relationship"? • What are patterns we see in Math? • How might patterns connect with relationships? • How might patterns connect with ratios? • What is a ratio? • What is a sequence? • How can a sequence help you identify a pattern? • How does being able to identify patterns help you solve problems?	• If we know the number of male and female bees in a generation, how do we find the number of parent bees? • When we calculate the number of bees in the next generation, what are we learning about patterns? • When we calculate bees in the next generation, what are we learning about relationships and ratios? • What patterns are emerging as you complete the table of the first twelve Fibonacci numbers? • What is the relationship these patterns represent? • How does this relate to our study of ratios and proportional reasoning? • How are ratios connected to patterns? • How are ratios connected to relationships? • What pattern develops as the value of the ratio changes as you complete the table? • How did you reach that conclusion? • Where can you find evidence of the golden ratio in our faces and bodies? • What is the relationship between your ratios and the golden ratio? • What is the relationship between your partner's ratios and the golden ratio? • If the ratios are different, why do you think this might be? • How can you use the patterns identified to generate ratios?	• What is the relationship between Fibonacci and patterns? • What do ratios tell us about patterns? • How can you use patterns to describe relationships? • What do you think would happen to the ratio if you skip every other Fibonacci number? • In what other places might you find the golden ratio? • What comparisons can you make between the ratios you and your partner found? • What have you learned about the connection between patterns, ratios, and relationships? • How can you use what you know about patterns and relationships to solve real world math problems? • How do patterns define relationships?

DIFFERENTIATION *(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.*			
Content	**Process**	**Product**	**Learning Environment**
This lesson is above grade level. The content of this lesson is unique for the curriculum.	Students will engage in in-depth critical thinking. The mathematical computation required in this lesson is advance.		
PLANNED LEARNING EXPERIENCES *(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)*			

Engage and Connect - This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.

Students will watch a clip from a PBS nature show about the life of honeybees. Students will learn that there are three types of honeybees: the drone bees, the worker bees, and the queen bee. They will also learn that unlike the human population, bee populations are disproportionately female. https://youtu.be/IE-8QuBDkkw

Bees and bee genetics can be an interesting way to explore some mathematical patterns and relationships.

Pre-Lesson Questions:

- What patterns in the video do you see about bees?
- What makes you identify them as patterns?
- Where are places we find or see patterns?
- What are some characteristics of patterns?
- What might patterns illustrate?
- What do you think about when I say "relationship"?
- What are some relationships we see in Math?
- How might patterns connect with relationships?
- How might patterns connect with ratios?
- What is a ratio?

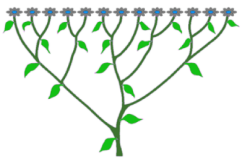
Students will work independently for approximately 8 minutes to complete a table to show a male bee at generation zero and determine how many ancestors he had in each previous generation one through five.

Questions:

- How can you describe the pattern that is emerging as you continue the generation chart?
- What patterns develop as you create your chart?
- What ideas that we have learned before were useful in completing this activity?
- What is the relationship this pattern might inform or define?
- How did you reach that conclusion?
- How could you prove that?

The pattern above is called the Fibonacci sequence. Some people think the Fibonacci sequence can be seen in nature.

Look at the three pictures, can you find the Fibonacci sequence in the pictures?



Questions:

- What patterns do you notice?
- What evidence is presented in the pictures that suggest the Fibonacci sequence exists in nature?
- What do you notice about the number of leaves and petals on the flowering plant in the picture?
- What do you wonder?

Explore - *In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.*

Students will work in groups of 4 and will use the knowledge they gained in the first part of the lesson to work together to write the first twelve Fibonacci numbers. Then they will calculate the ratio of each number to the previous one.

Questions:

- What patterns are emerging as you complete the table of the first twelve Fibonacci numbers?
- What is the relationship these patterns represent?
- How does this relate to our study of ratios and proportional reasoning?
- How is a ratio connected to patterns?
- How is a ratio connected to relationships?
- What happens to the value of the ratio as you complete the table?
- How did you reach that conclusion?

Explain - *Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.*

Students will watch a short video. ***https://youtu.be/yaovqXmQO1M***
In groups, students will use diagrams to show where DaVinci expected to find the golden ratio throughout the human body. Next, groups will work together to measure body parts. They will be instructed to measure the short length first. Then they will determine what the long length would need to be to be golden. After that, they will measure the actual long length and calculate the ratio of long : short. Where can you find the golden ratio in yourself?

Questions:
- How is patterning related to the golden ratio?
- What is the relationship between your ratios and the golden ratio?
- What is the relationship between your partner's ratios and the golden ratio?
- If the ratios are different, why do you think this might be?
- How can you use the patterns identified to generate ratios?

Elaborate —*Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways.*

Based on the investigations in previous phase, students will be asked:

- What did you notice about patterns and relationships as you were completing this activity?
- How would you describe the pattern identified in the activity?
- What connections did you make between patterns and ratios as you completed the activity?
- What connections did you make between patterns, ratios and the golden ratio as you completed this activity?
- In what other places might you find the golden ratio?

Evaluate - *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*

Students will be asked to work with a group mate to verify the ratios that were calculated.

Questions:
- What comparisons can you make between the ratios you and your partner found?
- What have you learned about the connection between patterns, ratios, and relationships?
- How can you use what you know about patterns and relationships to solve real world math problems?
- What is the relationship between Fibonacci and patterns?
- How can you use patterns to describe relationships?
- How do patterns define relationships?

TEACHER NAME			Lesson #
Kim Sholler			**3**
MODEL	**CONTENT AREA**	**GRADE LEVEL**	
Taba	Math	6 – 8	
CONCEPTUAL LENS		**LESSON TOPIC**	
Patterns		Fractals	

LEARNING OBJECTIVES *(from State/Local Curriculum)*

6.RP.1 Understand the concept of a ratio and use ratio language to: describe a ratio as a multiplicative relationship between two quantities and to model a ratio relationship using a variety of representations.
7.G Draw, construct, and describe geometric figures and describe the relationships between them.
8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

THE ESSENTIAL UNDERSTANDING ***(What is the overarching idea students will understand as a result of this lesson?***	**THE ESSENTIAL QUESTION** ***(What question will be asked to lead students to "uncover" the Essential Understanding)***
Patterns define relationships	How do patterns define relationships?
CONTENT KNOWLEDGE **(What factual information will students learn in this lesson?)**	**PROCESS SKILLS** **(What will students be able to do as a result of this lesson?)**
Students will know THAT… • A fractal is a mathematical shape that is infinitely complex. It is a pattern that repeats infinitely, and every part of the fractal regardless of how zoomed in, or zoomed out you are looks very similar to the whole image. • A pattern is a repeated design or recurring sequence. • Infinity is the idea of something that has no end. • A ratio describes a multiplicative relationship between two quantities. • A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, and dilations • Scaling up and scaling down a ratio maintains equivalency. Congruent figures have the same shape and the same size.	**Students will be able to…** • Analyze text • Synthesize information • Create generalizations • Provide evidence to support a claim • Work collaboratively • Come to a consensus • Establish and defend a position • Integrate Information • Manipulate information • Categorizing • Comparing and Contrasting

GUIDING QUESTIONS ***What questions will be asked to support instruction?*** ***Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding***		
Pre-Lesson Questions:	**During Lesson Questions:**	**Post Lesson Questions:**

• What does "pattern" mean to you? • How are patterns created? • What patterns do you see in nature? • What patterns do you see in math? • Where else do you see patterns? • What does "fractal" mean to you? • Where have you seen fractals? • What does "infinity" mean to you? • When is something infinite? • What is an example of something that is infinite? • What do you think might be the relationship between infinite and fractals? • What do you think might be the relationship between patterns and infinite? • What do you think might be the relationship between patterns and fractals?	• How could you group the fractal images? • Why would you group them that way? • How else could you group these images? • How could you label each group? • Which of these items could belong to more than one group? • Which of these items could be put into a different group? • Why would you group them that way? • What statement could you create that would always be true about patterns?	• What is the relationship between the patterns you noticed in the different fractal images? • How do patterns and relationships exist in nature? • How might climate changes effect the fractals we see in nature? • How are the words or phrases in the groups you created related to the concept, patterns and relationships? • How are the characteristics of fractals like that of patterns? • When I zoom in on the images what do you notice? • How do patterns define relationships?

DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	Learning Environment
Content and vocabulary is above grade level.	Students will engage in in-depth critical thinking. Students must analyze the concept of patterns through grouping and regrouping.	Students have choice in demonstrating their understanding of the concept/essential understanding.	Small group and whole group instruction. Phases of Taba are student led.

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect - This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.

Comment [AG3]: generalizing

To engage students, the teacher displays a series of images of fractals on the smartboard. The teacher asks:

1. What do you wonder about these images?
2. What do you notice about the images?
3. Where have you seen these images before?
4. How would you describe these images?
5. How are these images alike? How are they different?
6. What do these images have in common?

Teacher asks pre-lesson Questions: (please see above)

Students are asked to record notes about some of the characteristics of the images.
Teacher Hook: How are fractals like the air you breathe? Fractals are irregular repeating shapes that can be found in cloud formations, tree limbs, seashells, stalks of broccoli and rugged mountain ranges.

**A fractal is a never-ending, repeating pattern. It is an example of infinity. They are found in nature such as in snowflakes and ferns, the veins in a hand and the leaves in a tree, but are also used in a wide variety of very complicated engineering problems and fluid dynamics.

Students will watch a YouTube video about fractals. The video explains that fractals are complex, never ending patterns created by repeating mathematical equations. The video delves into the mysteries of fractals and their mysterious properties and how they can be found in nature.
https://youtu.be/DHNooAe44dY (first 3:29)

Explore - In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.

Listing: The teacher will instruct students to watch the video. While watching the video, they should list words or phrases that relate to patterns in some way. Next, the teacher will record student findings on the whiteboard.

Explain - Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.

2. Grouping and Labeling:
Grouping: Teacher will divide students into groups of 4 based on a quick analysis of their ability to identify complex patterns in the images. Students are asked to choose no less than 20 words that are related to some aspect of patterns, from the class list to work with in grouping. The teacher will ask which words and/or phrases belong together because they are related in some way? Students will work in their small groups to group the words or phrases that belong together from the list on the whiteboard.

Expectations:

1. There must be at least three different groups.
2. Each group must have at least two words or phrases
3. No word can be used more than once; member of only one group

Labeling: Students give labels for the newly defined groups. Students will explain the reasons for their choices. The teacher will ask, "What would you call these groups you have formed?" and "Why would you group them together?" The students will verbalize a label(s) that encompasses all items in a group and will identify and verbalize common characteristics of items in a group.

The teacher will move throughout the classroom checking in and monitoring student groups. As students finish, the teacher will ask students to label the groups to indicate how the words or phrases in the group are related. Students will be able to explain their mathematical thinking to the teacher by describing the similarities and differences among the groups. The teacher will ask the student to defend their choice of label and the reasons for the words being grouped as they are. Students will share their labels and reasons for grouping with the whole class.

The Teacher will ask during-lesson questions after students have finished grouping:

4. How could you group the fractal images?
1. Why would you group them that way?
2. How else could you group these images?
3. How could you label each group?
4. Which of these items could belong to more than one group?
5. Which of these items could be put into a different group?
6. Why would you group them that way?

Elaborate: Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways

3. Subsuming, Regrouping, Renaming:
Student groups will be challenged to regroup items. Subsuming if they are able (one group merges with another group). The new groups must be new categories with new labels.

Expectations:

1. Items can be used in more than one category (group)
2. New labels must be assigned for the categories (groups)
3. Groups must have at least three words per group since (words can be used in multiple groups)
4. Categories must be based on some aspect of the concept, "patterns and relationships"

Teacher will ask all groups to share their categories (labels with some examples of words).

Evaluate: *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*

Generalizing:
Based on the learning experience, discussion and sharing, students will work with their groups to answer the following question, "What is a generalization you could write that would always be true about patterns?" All groups will present their generalizations to the class. After presentations, the whole group votes on the best generalization. The best generalization will be the one that supports the essential understanding that patterns define relationships.

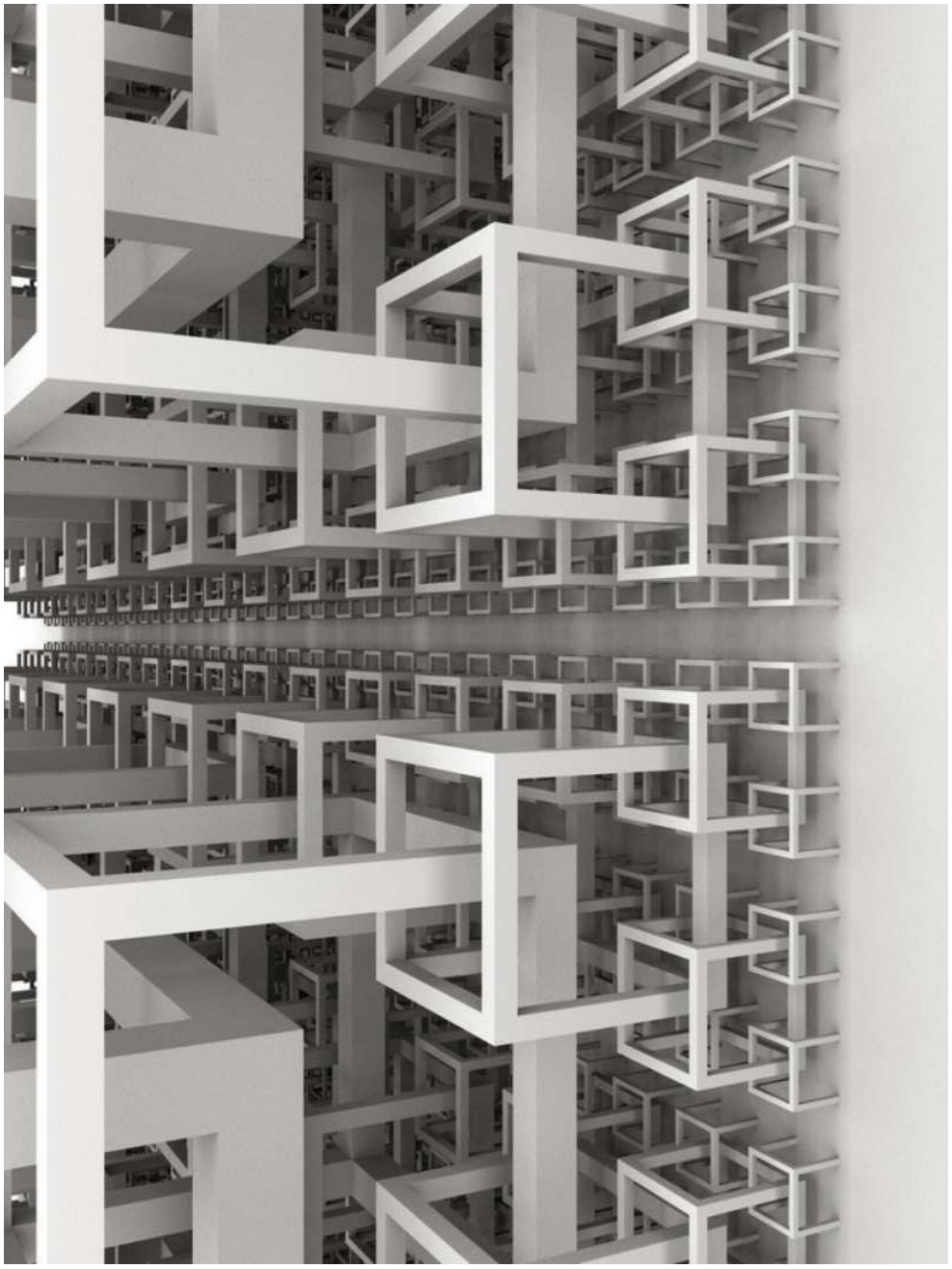
The teacher asks the post-lesson questions:

1. What is the relationship between the patterns you noticed in the different fractal images?
2. How do patterns and relationships exist in nature?
3. How might climate changes effect the fractals we see in nature?
4. How are the words or phrases in the groups you created related to the concept, patterns and relationships?
5. How are the characteristics of fractals like that of patterns?
6. When I zoom in on the images what do you notice?
7. How do patterns define relationships?

Students will be given time to play with different types of fractals before being asked to create their own. Students will select one of the fractal explorations to learn about and write a short journal entry about what then learned. Then the students select one or more types of fractals to draw or create digitally.

Images for Engage and Connect:


GVK
2



TEACHER NAME			Lesson #
Kimberly Sholler			4
MODEL	**CONTENT AREA**	**GRADE LEVEL**	
VTS	Math	Middle School	
CONCEPTUAL LENS		**LESSON TOPIC**	
Patterns		Fractal Geometry	

LEARNING OBJECTIVES *(from State/Local Curriculum)*

6.G.1 Create geometric models to solve real-world and mathematical problems
7.G.1 Solve problems involving scale drawings of geometric figures

THE ESSENTIAL UNDERSTANDING *(What is the overarching idea students will understand as a result of this lesson?*	**THE ESSENTIAL QUESTION** *(What question will be asked to lead students to "uncover" the Essential Understanding)*
Patterns define relationships	How do patterns define relationships?

CONTENT KNOWLEDGE (What factual information will students learn in this lesson?)	**PROCESS SKILLS** (What will students be able to do as a result of this lesson?)
Students will know that: • A fractal is a mathematical shape that is infinitely complex. It is a pattern that repeats infinitely, and every part of the fractal regardless of how zoomed in, or zoomed out you are looks very similar to the whole image. • A pattern constitutes a set of numbers or objects in which all the members are related with each other by a specific rule or relationship. • A pattern is also known as a sequence. • There can be a finite or infinite number of items in a pattern. • Finite is a set of numbers or objects that can be counted. • Infinite is the idea that something has no end. • Growing patterns increase by a constant amount. • Patterns can consist of sounds, movements, objects or numbers. • A paradox is a something that may be true but seems impossible or difficult to understand because it contains two opposite facts or characteristics. • Self-similarity is when an object is a copy of itself on a different scale. • Patterns are used to represent identified regularities. • Patterns are used to form generalizations. • Patterns allow us to see relationships and develop generalizations. • Patterns are used to analyze data and provide a sense of order.	Students will be able to: • Analyze • Explain • Infer • Access, organize and synthesize information • Create generalizations • Provide evidence to support a claim • Work collaboratively • Come to a consensus • Establish and defend a position • Integrate Information • Manipulate information • Discuss and debate at high levels

GUIDING QUESTIONS

What questions will be asked to support instruction?
Include both "lesson plan level" questions as well as questions designed to guide students to the essential understanding

Pre-Lesson Questions:	During Lesson Questions:	Post Lesson Questions:
• What do you think about when you hear the word patterns? • What is a pattern? • Where can see patterns? • Where can you hear patterns? • What does it mean when something is finite? • What is an example of something that is finite? • What does it mean when something is infinite? • What is an example of something that is infinite? • What is a fractal? • Where can you find examples of fractals? • How can you connect fractals to patterns? • What relationship can you identify between fractals and patterns? • How do patterns define relationships in fractals illustrated in the video? • How can you use patterns to form generalizations? • What relationships are defined by fractals? • What relationships are defined by patterns? • How would you define the relationship between fractals and patterns?	• What do you see? • What do you see in this image? • What do you see that makes you say that? • What else do you see? • What is going on in the image? • What do you see that makes you say that? • What else is going on in the image? • What do you see that makes you say that? • What patterns do you see in the image? • What shapes do you see in the image? • What is the relationship between the shapes and the patterns in the images? • Who agrees with that observation? • Who disagrees with that observation? • Why do you think that? • What could you add to that observation? • What is the relationship between the patterns you notice and the fractal image that is created? • How does the image represent a fractal?	• What do you see? • What do you see that makes you say that? • What else do you see? After several rounds, questions include: • What do you see that represents a pattern? • What is the relationship between that pattern and the fractal image? • What makes you say that? • What do you see that relates to things you may find in nature? • What makes you say that? • What do you see that relates to things you may find in a piece of artwork? • What makes you say that? • What do you see that relates to problems you may need to solve in math? • What makes you say that? • What patterns are represented in your image? • How do the patterns represented in your image create a fractal image? • How do patterns define relationships?

DIFFERENTIATION

(Describe how the planned learning experience has been modified to meet the needs of gifted learners. Note: Modifications may be in one or more of the areas below. Only provide details for the area(s) that have been differentiated for this lesson.

Content	Process	Product	Learning Environment
This lesson is above grade level. Some images used in this lesson are unfamiliar to students and may express/illustrate concepts, which are not usually a part of the curriculum at this grade level.	VTS is an open-ended thinking and inquiry strategy, which allows students to share unique perspectives and make inferences. Students articulate meaning they make from visual images.		This is a student led lesson.

PLANNED LEARNING EXPERIENCES

(What will the teacher input? What will the students be asked to do? For clarity, please provide detailed instructions)

Engage and Connect - This phase focuses on piquing students' interest and helping them access prior knowledge. This is the introduction to the lesson that motivates or hooks the students.

As students enter the room, Fractals in Nature, by Sladjana Trajkovic, illuminates the interactive whiteboard. The teacher begins the lesson by asking the pre-lesson questions. https://www.youtube.com/watch?v=4IRLvYOZD8A

Pre-Lesson Questions:

What do you think about when you hear the word patterns?

- What is a pattern?
- Where can see patterns?
- Where can you hear patterns?
- What does it mean when something is finite?
- What is an example of something that is finite?
- What does it mean when something is infinite?
- What is an example of something that is infinite?
- What is a fractal?
- Where can you find examples of fractals?
- How can you connect fractals to patterns?
- What relationship can you identify between fractals and patterns?
- How do patterns define relationships in fractals illustrated in the video?
- How can you use patterns to form generalizations?
- What relationships are defined by fractals?
- What relationships are defined by patterns?
- How would you define the relationship between fractals and patterns?

Students will be given a sheet of paper (plain or graph paper). They are instructed to create a fractal image. As they create their image, they are instructed to think about the patterns in their image and where else these patterns may be found in math, nature, architecture, etc.

Once students have created their image (20 minutes), teacher facilitates a whole group conversation. Students are asked to share their unique drawings. As students share their drawings, the teacher asks:

- What pattern(s) are repeating in your drawing?
- Why did you choose to use that pattern?
- What can you tell us about the relationship between the shape of your drawing and the patterns you used to create it?
- What makes your drawing an example of a fractal image?
- If I were to fold your paper in half, what would happen to your fractal image? Would this change the relationship between the pattern and the fractal image?
- If I were to put your drawing under the document camera an enlarge it, what would happen to your fractal image? Would this change the relationship between the pattern and the fractal image?

Explore - *In this phase, the students have experiences with the concepts and ideas of the lesson. Students are encouraged to work together without direct instruction from the teacher. The teacher acts as a facilitator. Students observe, question, and investigate the concepts to develop fundamental awareness of the nature of the materials and ideas.*

Students will be asked to look at the interactive whiteboard. Students will be instructed to look at the image displayed for 3 minutes in silence. After 3 minutes of focused observation, the teacher asks students:

- What do you see in this image?
- What do you see that makes you say that?

The first two questions are asked of individual students for multiple rounds. Allow each student who volunteers to answer both questions. The teacher may paraphrase what the student responds. The teacher may point to the space or object in the picture to which the student is referring. The teacher gives no opinion or judgement about what the student says.

Comment [AG4]: Yes/no? NO! Please rephrase: How would ….

After several rounds when the discussion wanes, the teacher will ask students:

- What else do you see?
- What is going on in the image?
- What do you see that makes you say that?
- What else is going on in the image?
- What do you see that makes you say that?

Now that several rounds of the foundational observation questions have occurred, students will be asked:

- What patterns do you see in the image?
- What shapes do you see in the image?
- What is the relationship between the shapes and the patterns in the images?
- Who agrees with that observation?
- Who disagrees with that observation?
- Why do you think that?
- What could you add to that observation?

Students answer the questions:

What patterns do you notice? What is the relationship between the patterns you notice and the fractal image that is created? How does the image represent a fractal?

Explain - *Students communicate what they have learned so far and figure out what it means. This phase also provides an opportunity for teachers to directly introduce a concept, process, or skill to guide students toward a deeper understanding.*

After students respond to the question above, the teacher will play the following YouTube video:
https://www.youtube.com/watch?v=2kZv22Ip1Vs

Questions:
- What did you notice about the triangle image in the video?
- Why is that a paradox?
- What relationships did you identify as they zoomed in on the triangle image?
- What makes this object self-similar?
- How can we apply what we know about fractal geometry to other subjects or disciplines?
- What common characteristics do all of the images in the video share?
- How does fractal geometry define relationships?
- What mathematical patterns and relationships can you identify?

Students are divided into small groups for the next learning experience.

Elaborate —*Allow students to use their new knowledge and continue to explore its implications. At this stage students expand on the concepts they have learned, make connections to other related concepts, and apply their understandings to the world around them in new ways.*

Each small group of students will select an image for analysis. Images will be numbered, students draw numbers to see which image they will use for this learning experience. Students will work in small groups. Each small group will use VTS strategies.

Questions:
- What do you see?
- What do you see that makes you say that?
- What else do you see?

After several rounds, questions include:

- What do you see that represents a pattern?
- What is the relationship between that pattern and the fractal image?
- What makes you say that?
- What do you see that relates to things you may find in nature?
- What makes you say that?
- What do you see that relates to things you may find in a piece of artwork?
- What makes you say that?
- What do you see that relates to problems you may need to solve in math?
- What makes you say that?

Each group, based on their discussion answers the following questions:

What patterns are represented in your image?
How do the patterns represented in your image create a fractal image?

Evaluate - *This phase assesses both learning and teaching and can use a wide variety of informal and formal assessment strategies.*

Each group presents their image and the following information:

- The name of the image or some identification for the image.

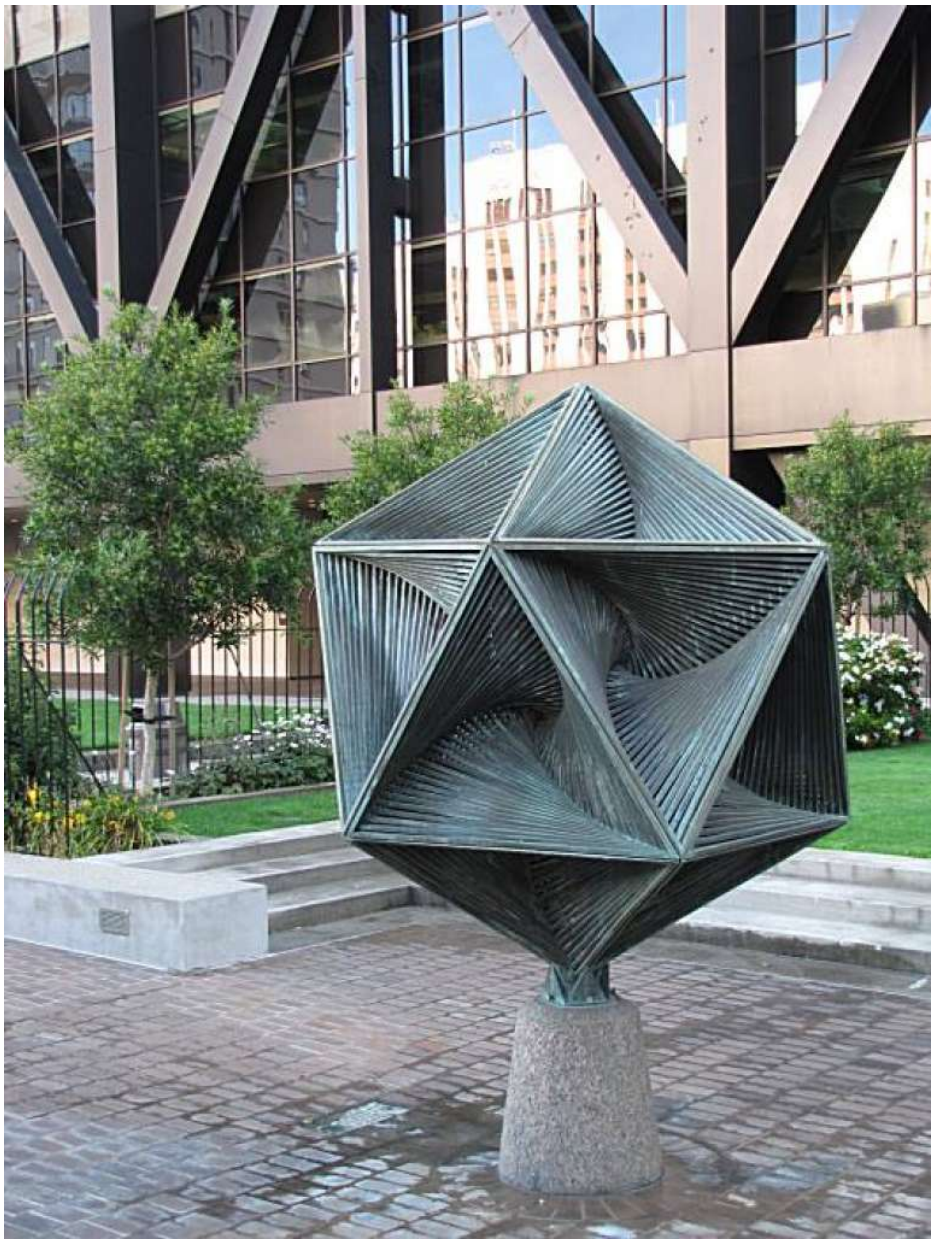
They also present answers from these questions as other students listen and take notes on a graphic organizer:

- What patterns are represented in your image?
- How do the patterns represented in your image create a fractal image?

Final Assessment:
Based on the images we have seen today, how would you respond to the following question:

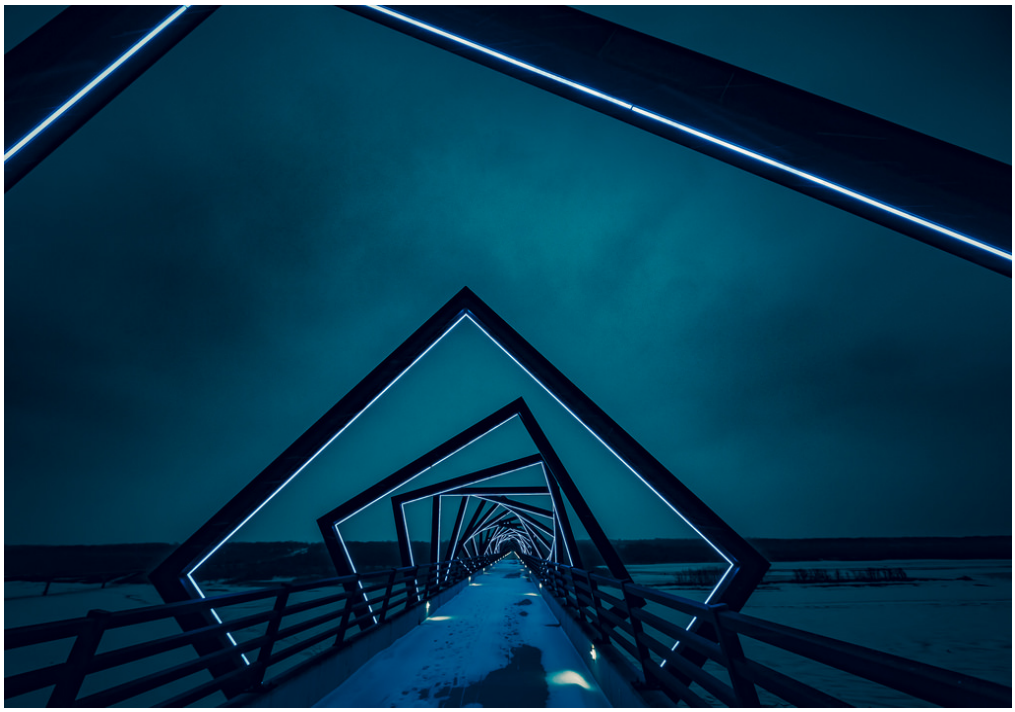
How do patterns define relationships?

Image for Explore Section:

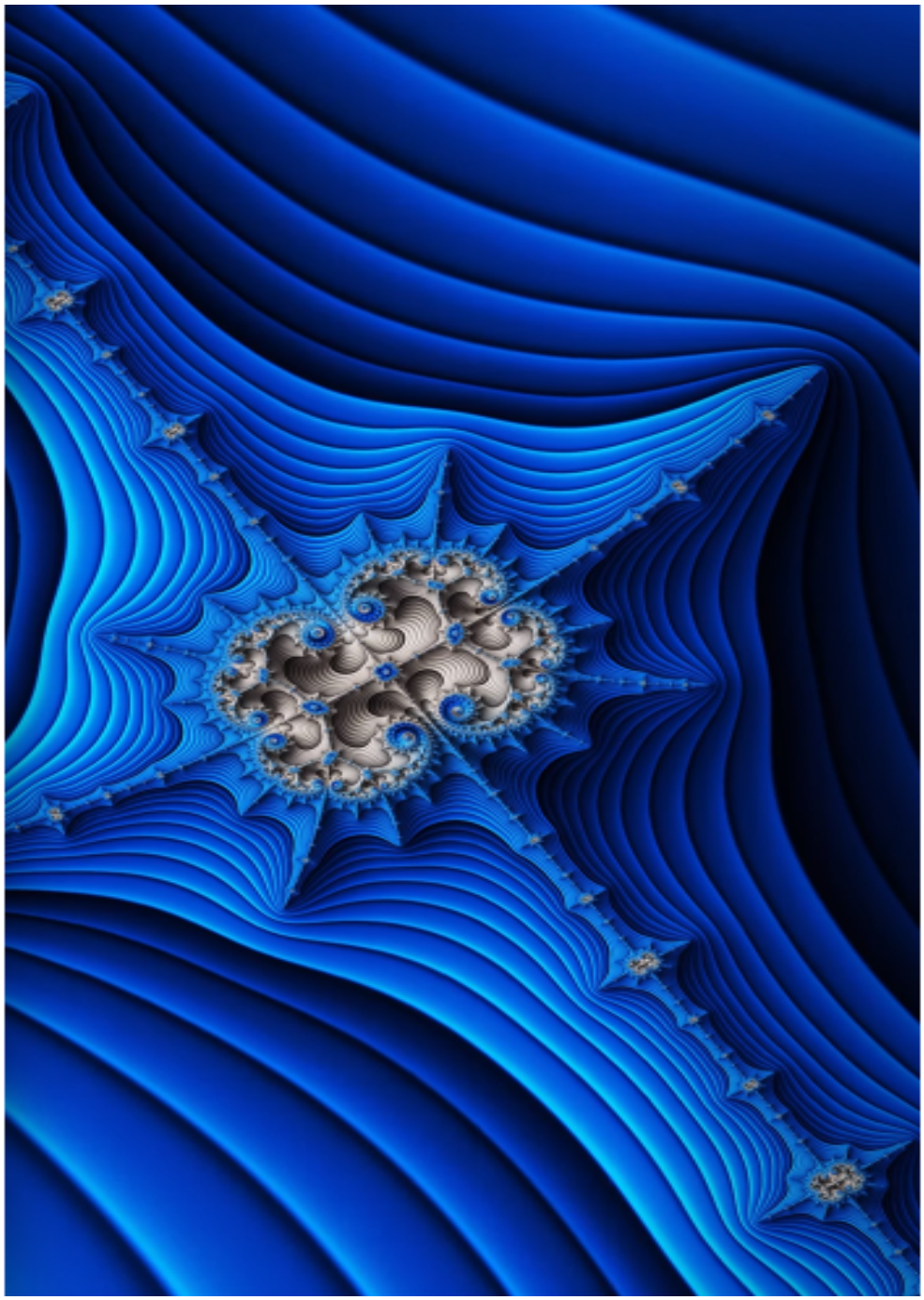


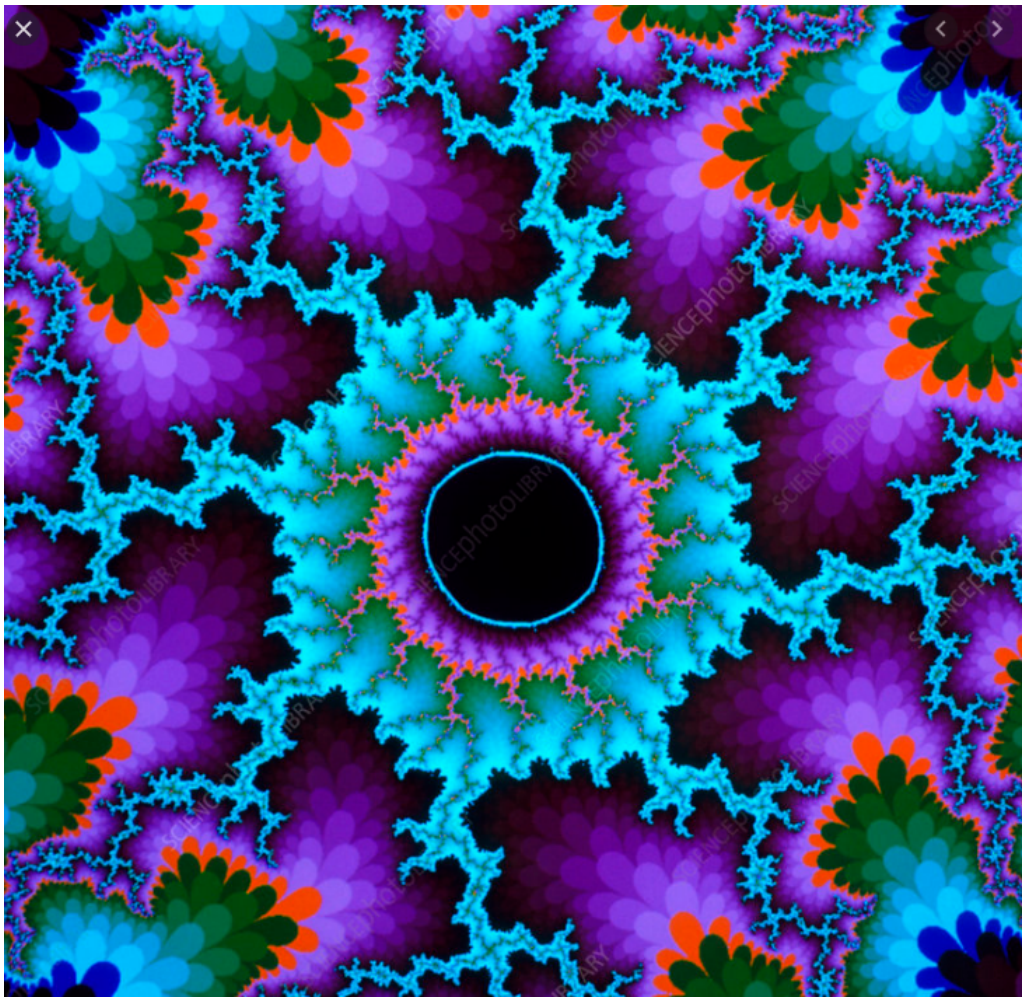
Images for the Elaborate Section:

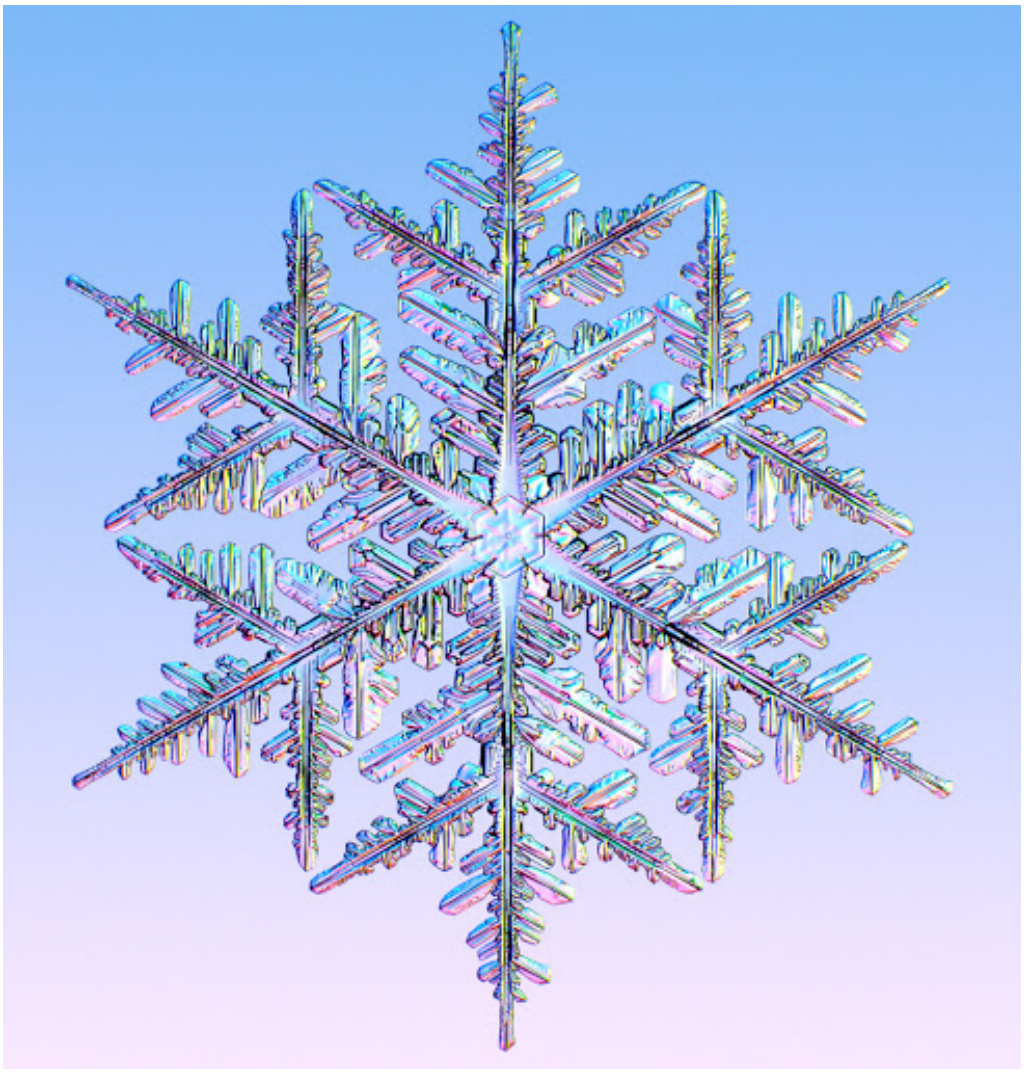
Twisted bridge, Vlaardingen, Netherlands

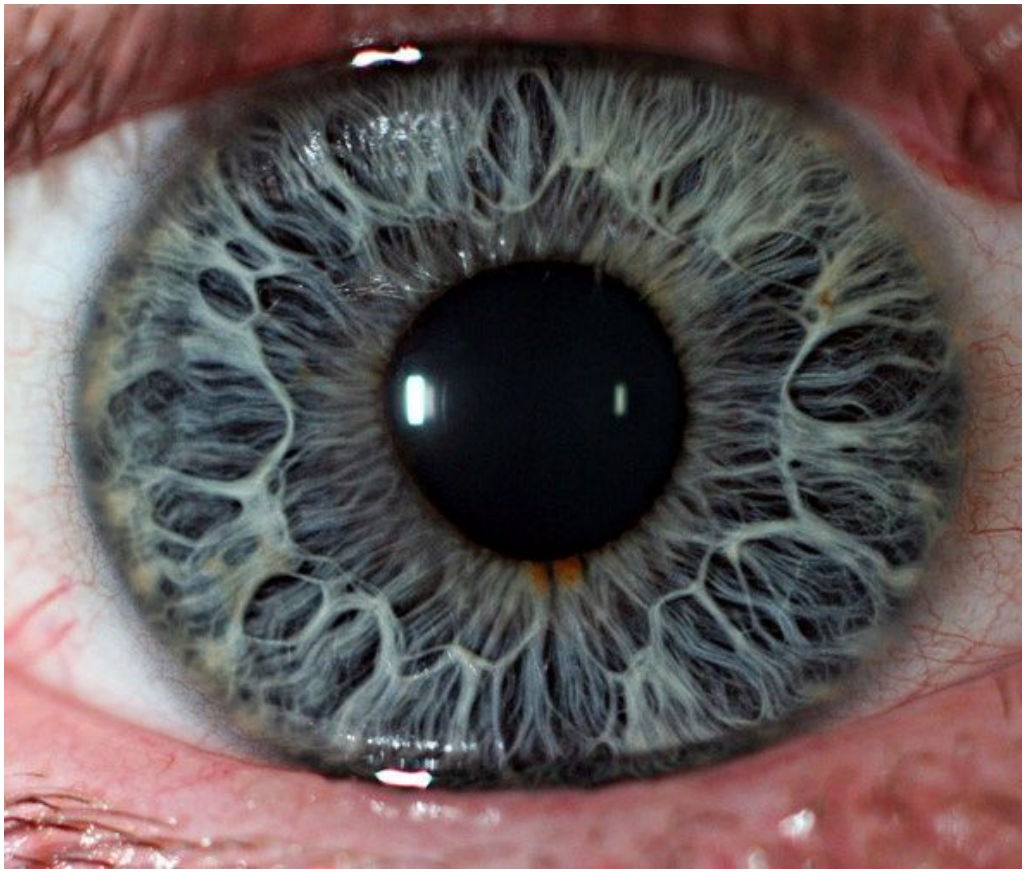



Vitor Estrela Santos
Vitor Estrela Santos









V Unit Resources

Articles and Books

Stephens, K.R. & Karnes, F.A. (Eds.) (2016). Introduction to Curriculum Design in Gifted Education. Waco, TX: Prufrock Press

This resource provides an in-depth look at curriculum development for gifted students. It provides the general foundations of good curriculum design and it provides information about designing curriculum across content areas and the role of assessment in the curriculum design process. This is an excellent resource.

Erickson, H. L. (2007). Concept-based curriculum and instruction for the thinking classroom. Thousand Oaks, CA: Corwin Press

This is a great resource that combines curriculum design with teaching methods encourages students to learn concepts as well as content and skills. Ideas from this resource were used as I designed this unit.

Wiggins, G. P., & McTighe, J. (2011). The Understanding by Design Guide to Creating High-Quality Units. Alexandria, VA: Association for Supervision and Curriculum Development

This is a great resource that provides information on “backwards design”. It was an incredibly helpful resource while planning this unit.

Azzam, A. (2016, April). Six Strategies for Challenging Gifted Learners. Retrieved April 23, 2019, from http://www.ascd.org/publications/newsletters/educationupdate/apr16/vol58/num04/Six-Strategies-for-Challenging-Gifted-Learners.aspx

This resource includes strategies and ideas that can be used to develop curriculum and lessons for gifted students. Ideas from this article were used to create this curriculum.

ReadWorks Article: Mr. Max the Math Teacher

This article is utilized for the Taba lesson in this unit. It presents ideas about how patterns in behavior can define relationships. Students are asked to reflect on the reading and to either compose an essay or create a video or skit demonstrating what they learned and how the patterns and relationships in their lives inform their decision-making.

Video Links:

"Investing in our Youth: Math is Everywhere" https://youtu.be/Hh1M409ed1I

This is an introductory video used to hook the students during the Team Based Learning lesson. It will help students begin to think about patterns and relationships.

"Mathematics in Real Life" https://youtu.be/dpv06SFHtRg

This video is part of the TBL Lesson. It show how math is used in everyday life. This video will help students make connections between math, patterns, relationships and real world applications.

"Music and Math: The genius of Beethoven https://youtu.be/zAxT0mRGuoY

This video part of the TBL lesson and is about how Beethoven was able to become one of the most significant composers of all time, even though he wrote his most beloved songs while going deaf. The answer lies in the math behind the music.

"Nature – Silence of the Bees inside the Hive PBS" https://youtu.be/lE-8QuBDkkw

This video is used in the Questioning lesson and is an up-close look at a honeybee hive, including the labors of the queen, worker bees and drones. This will help students make a connection between bees and bee genetics to explore mathematical patterns and relationships.

"Leonardo Da Vinci 500 years of Genius" https://youtu.be/yaovqXmQ01M

This video is used in the Questioning lesson and is about DaVinci and where he expected to find the golden ratio throughout the human body. Students will use what they learned from the video to try to find examples of the golden ratio in their own body.

"Fractals: a world in a grain of sand" https://youtu.be/DHNooAe44dY

This video is used in the Taba lesson. This video is about fractal imagery. It begins by looking at the human body. Our lungs manage to pack the surface area of a tennis court into our ribcage. Our circulatory system crams 60,000 miles of branching blood cells and capillaries into our bodies. Single objects repeating patterns over and over and over at various scales, parts similar to the whole. These examples of fractals in action are presented in dazzling detail by a mathematician by name of Ben Weiss.

"Fractals in Nature" https://www.youtube.com/watch?v=4IRLvYOZD8A

This is a short video used in the VTS lesson that examines fractals in nature. It is used as the hook in the VTS lesson. It explains that fractals are complex, never ending patterns created by repeating mathematical equations. The video delves into the mysteries of fractals and their mysterious properties and how they can be found in nature. This will provide inspiration as students are asked to think about and create a fractal image using patterns.

"What is Fractal Geometry – A Breakthrough Junior Challenge"
https://www.youtube.com/watch?v=2kZv22Ip1Vs

This video is used in the VTS Lesson and explores the mystery of fractal geometry. The students will use the information from this video to help them identify geometric patterns in images.

Fractal Images were found on the following websites:

The Fractal Foundation: https://fractalfoundation.org/resources/what-are-fractals/#:~:text=Fractals%20are%20infinitely%20complex%20patterns,systems%20%E2%80%93%20the%20pictures%20of%20Chaos.

Mathigon: https://mathigon.org/course/fractals/introduction

The Fractal Forge: https://thefractalforge.com/what-is-a-fractal/

Fractals in Nature: https://www.diygenius.com/fractals-in-nature/#:~:text=A%20fractal%20is%20a%20pattern,the%20biodiversity%20of%20a%20forest.